Introduction to Econometrics

Fall 2006

Assignment 5

Today's Date: 10/16
Due Date: 10/23

Please show all of your work and clearly indicate your final response to each question. These are essentially practice questions for the midterm, so answer the questions carefully and succinctly in trying to simulate some of the time constraints you will face. I would also advise you to work by yourself as much as possible, at least in your initial attempts to answer the questions.

1. Consider the linear bivariate regression model

$$y = \beta_0 + \beta_1 x + \varepsilon.$$

Assume that the

$$\begin{aligned} E(\varepsilon|x) &= 1, \\ Var(\varepsilon|x) &= 2, \text{ for all } x. \end{aligned}$$

You have access to a random sample of size N containing the information $\{y_i, x_i\}_{i=1}^N$.

   1. Consider the estimator

   $$\hat{\beta}_1 = \frac{\sum_{i=1}(y_i - \overline{y})(x_i - \overline{x})}{\sum_{i=1}(x_i - \overline{x})^2},$$

   where $\overline{x}$ and $\overline{y}$ are the sample means of x and y. Is $E(\hat{\beta}_1) = \beta_1$?

   2. Consider the estimator

   $$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}.$$

   Is $E(\hat{\beta}_0) = \beta_0$? If not, compute the bias of this estimator.

2. Consider the multiple regression model

$$y = \beta_0 + \beta_1 z_1 + \beta_2 z_2 + \varepsilon,$$

where

$$\begin{aligned} E(\varepsilon|x, z) &= 0 \\ Var(\varepsilon|x, z) &= \sigma^2 z_1^4. \end{aligned}$$

1. Is the OLS estimator of β unbiased?
2. Write down the GLS estimator of β by first transforming the model so that the "new" disturbance term has a common variance across all observations. That is, find a transformation $T(x, z)$ such that the disturbance term in the equation

$$T(x, z)y = T(x, z)Z\beta + T(x, z)\varepsilon$$

is mean independent and has constant variance.
3. Show that the GLS estimator is simply the OLS estimator applied to the data in the transformed equation above.

3. Consider the bivariate linear regression

$$y = \beta_0 + \beta_1 x + \varepsilon,$$

where ε is independent and identically distributed as a normal random variable with mean 0 in the population. From a random sample of $N = 15$ observations on y and x, you find that $\sum_{i=1}^{N} x_i = 0$ and $\sum_{i=1}^{N} x_i y_i = 0$. Do you have enough information to determine the probability that $\beta_1 > 0$ in the population? If so, what is the probability?

4. A linear model consists of single equation, where both y and x are expressed as deviations from their respective sample means:

$$y = \beta x + \varepsilon.$$

The disturbance term ε is independently and identically distributed (i.i.d.) with variance σ_ε^2. While you have access to a random sample with perfect measurements on y, x is measured with error, where

$$x^* = x + u$$

is the measure to which you have access, where u is i.i.d. with mean 0 and variance σ_u^2. You have access to a random sample of observations on $\{y_i, x_i^*\}_{i=1}^{N}$.

1. Is the estimator

$$\hat{\beta} = \frac{\sum_{i=1}^{N} y_i x_i^*}{\sum_{i=1}^{N} (x_i^*)^2}$$

an unbiased estimator of β? Show why or why not.

2. Imagine you have access to another variable for each sample member, z_i. In the population,

$$\begin{aligned} Exz &\neq 0 \\ Ez\varepsilon &= 0 \\ Ezu &= 0. \end{aligned}$$

Consider the estimator

$$\tilde{\beta} = \frac{\sum_{i=1}^{N} y_i z_i}{\sum_{i=1}^{N} x_i z_i}.$$

Determine whether $\tilde{\beta}$ is a consistent estimator of β, i .e., if $\operatorname{plim} \tilde{\beta} = \beta$.

5. Consider the regression function

$$d = X\beta + \varepsilon,$$

where d is a $N \times 1$ vector of binary variables, X is a $N \times K$ matrix of values of exogenous variables, and ε is a vector of disturbances with the property $E(\varepsilon|X) = 0$.

1. Is $\hat{\beta} = (X'X)^{-1}X'd$ an unbiased estimator of β?
2. Write down, in detail, a consistent estimator for the covariance matrix of β. Are there any practical difficulties you may encounter in estimating this matrix? If so, propose an alternative consistent estimator of the covariance matrix of $\hat{\beta}$.