Introduction to Econometrics

Final Examination

Fall 2004

Please answer all of the questions and show your work. If you think a question is ambiguous, clearly state how you interpret it before providing an answer.

1. (18 points) A time series regression model is specified as

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t, \ t = -\infty, ..., 0, ..., \infty.$$

The disturbance term follows a first order autoregressive process,

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t,$$

where u_t is independently and identically distributed with mean 0 and variance σ_u^2, and where the parameter ρ is less than 1 in absolute value. You have access to a sample of observations on $\{y_s, x_s\}_{s=1}^T$. Define consistent estimators of β_0, β_1, ρ, and σ_u^2. Make sure that you provide a concise argument as to why the estimators you propose are consistent.

2. (20 points) Consider the panel data regression model

$$y_{it} = \beta_0 + \beta_1 x_{it} + \varepsilon_{it}, \ i = 1, ..., I; t = 1.2;$$

where y_{it} is individual $i's$ value of the dependent variable at time t and x_{it} the is the value of the regressor for individual i at time t. The disturbance term is given by

$$\varepsilon_{it} = m_i + u_{it},$$

where u_{it} is independently and identically distributed (i.i.d.) with mean 0 and variance σ_u^2.

   (a) If m_i is i.i.d. in the population with mean 0 and variance σ_m^2, define a consistent and (asymptotically) efficient estimator of β.

   (b) Say that m_i is not independently distributed with respect to x_{it}. Furthermore, you are unsure as to the nature of the dependence between m_i and x_{i1} and $x_{i2.}$ Define unbiased estimators of β_0 and β_1 in this case, if such estimators exist.

3. (17 points) Define a latent variable

$$y_i^* = \beta z_i + u_i,$$

where u_i is independently and identically distributed as a uniform random variable on the interval $[-.5, .5]$. You have access to a random sample of N observations $\{d_i, z_i\}_{i=1}^N$, where $d_i = 1$ if $y_i^* > 0$ and $d_i = 0$ if not. Define an unbiased Least Squares estimator of β, if one exists.

4. (27 points) Unemployment spells have a negative exponential distribution in two separate populations (A and B), that is,

$$\begin{aligned} f^j(t) &= \alpha_j \exp(-\alpha_j t), \\ F^j(t) &= 1 - \exp(-\alpha_j t), \end{aligned}$$

where $t \geq 0$ and $\alpha_j > 0, \ j = A, B$.

(a) You have access to a random sample of N observations from population A, $\{t_i\}_{i=1}^N$. The mean value of t in the sample is 5. What is the maximum likelihood estimate of α_A?

(b) You have access to another random sample of N unemployment durations from population B. In this sample, the duration information that is available is

$$t^* = \begin{cases} t & \text{if} \quad t < 12 \\ 12 & \text{if} \quad t \geq 12 \end{cases},$$

that is, all durations that are greater than 12 months are coded as being 12 months, while all those that last less than 12 months are measured exactly.

i. Write down the log likelihood function for this sample, where the dependent variable is t^*. Let p_c denote the proportion of sample members with censored observations, which are values of t^* that are equal to 12.

ii. The sample mean of t^* in this sample is 4, and $p_c > 0$. Is it possible that the maximum likelihood estimate of α_B is equal to α_A? Provide a rigorous justification for your answer.

5. (18 points) Say that you want to estimate the bivariate regression model

$$y = \delta x + \varepsilon,$$

where y is a random variable with mean 0, x is a mean zero random variable, and ε is independently and identically distributed with mean 0 and variance σ_ε^2. You have access to the measures

$$\begin{aligned} y^* &= y + u, \\ x^* &= x + v, \end{aligned}$$

where u is measurement error in the dependent variable that is i.i.d. with mean 0 and variance σ_u^2 and v is measurement error in the independent variable that is i.i.d. with mean 0 and variance σ_v^2. We are considering the regression equation

$$y^* = \beta x^* + w,$$

and the OLS estimator of β, which is

$$\hat{\beta} = \frac{\sum_{i=1}^{N} y_i^* x_i^*}{\sum_{i=1}^{N} (x_i^*)^2}.$$

(a) If $\sigma_u^2 > 0$ and $\sigma_v^2 = 0$, then $\hat{\beta}$ is a consistent estimator of δ (True or False, with justification).

(b) If $\sigma_u^2 = 0$ and $\sigma_v^2 > 0$, then $\hat{\beta}$ is a consistent estimator of δ (True or False, with justification).