Consider an agent living at time 1 and 2. Denote his consumption at
time $t$ by $c_t$. Let the agent’s intertemporal preferences be:

$$\ln c_1 + \beta \ln c_2, \quad 0 < \beta < 1$$

Let the agent face an interest rate $1 + r$, and income $m_1$ and $m_2$ at times 1 and 2, respectively.

1. Write the agent’s intertemporal budget constraint.

2. Derive a solution for the agent’s intertemporal choice problem in terms
   of $c_1$, $c_2$ and $S = m_1 - c_1$.

3. How does $S$ depend on $r$? Answer analytically; that is, compute $\frac{dS}{dr}$
at the solution for $S$, and sign it.

4. Suppose there economy is composed of one agent with income $m_1 = 0,$
   $m_2 > 0$; and one other agent with income $n_1 = m_2 > 0$ and $n_2 = 0.$
   Can you derive the equilibrium interest rate $1 + r$?