Question 1 (30 points)

Consider a two period economy. Agents are all identical, that is, there is one representative agent. The representative agent is alive at time $t$ and $t + 1$, and has preferences:

$$lnx_t + \beta lnx_{t+1}, \beta < 1.$$ 

This agent is endowed with 10 units of the consumption good at time $t$ and at time $t + 1$. There is no inflation in this economy, and hence you can assume throughout that the price for the good at time $t$ is 1.

1. Write down the consumer maximization problem of the representative agent (call the real interest rate $r$ in the budget constraint), first order conditions, and demand functions.

2. Write down the market clearing conditions (that is, feasibility conditions) for the whole economy.

3. Solve for the equilibrium interest rate and for the representative agent equilibrium allocation.

4. Suppose the agent cannot borrow and lend, that is, savings are zero and there is no interest rate $r$: he/she has to consume his/her own endowment in each period. How would you write the budget constraints? [hint: the plural is not a typo; there is a budget constraint for each time period] Once again the price of consumption at $t$ and also at $t + 1$ now can be normalized to 1. Are his/her equilibrium allocations changed?

Suppose again that the representative agent cannot borrow or lend, but he/she can now invest units of the consumption good at time $t$ (he/she still has his/her endowment as before). Call the amount invested $k_{t+1}$; the production function is

$$y_{t+1} = \alpha k_{t+1}, \alpha > 1$$
that is, the agent can give up \( k_{t+1} \) units of consumption at time \( t \) to get \( \alpha k_{t+1} \) extra units at time \( t + 1 \) (this is all per-capita; in terms of the notation used in class, \( k_t = 0 \)). Assume \( \alpha \beta > 1 \) (you will need this later).

1. Write down the budget constraints for the representative agent \([be careful here! the plural is still not a typo]\).

2. Write down the consumer maximization problem, using the budget constraints you derived, to solve for the optimal choice of investment \( k_{t+1} \). Then solve for the equilibrium allocations.

3. \([Bonus Question; this is hard]\) Suppose that borrowing and lending markets are now open, that is, there is an interest rate to be determined. This of course together with the investment and production technology as above. How does the budget constraint \([yes, singular]\) look like? Are equilibrium allocations changed? What is the equilibrium interest rate?

\textbf{Question 2 (20 points)}

Consider an economy with 2 states of the world, either ”a tax cut is voted” or ”a tax cut is not voted.” The tax cut is voted with probability .3. Consider a financial market with 2 basic assets. The first pays 2 dollars if the tax cut is not voted and 3 if it is. The second asset pays nothing if the tax cut is not voted and 1 dollar if it is. The prices of the two assets are, respectively 3 and 1 dollars. There is also another asset, a risk free bond that pays 10 dollars no matter if the tax cut is voted or not. This last asset sells for 9 dollars.

1. Is there an arbitrage opportunity in this economy?

2. Does your answer depend on the probability of the tax cut? Explain why.

3. Just for the fun of it, compute the variance of the two basic assets, and their covariance.