Political Economy of Centralized Redistribution and Local Government Fiscal Structure

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Abstract

This paper is concerned with centralized redistribution and local public good provision in an economic federation with two levels of government. The central government (or state) redistributes via a distortionary income tax and lump-sum transfer to all households. Multiple local governments use property taxes on housing consumption to finance local public goods. The central government’s fiscal policy is determined first through a majority vote of all households. Then the residents of each community vote on their respective local government’s fiscal policy. When voting on the central government’s fiscal policy, households are able to predict the impact of centralized redistribution on the allocation of households across the local communities and on local housing prices and public good provisions. When households vote in their local communities on the property tax rate and the public good provision they are constrained by housing market distortions and by the ability of people to freely move from one jurisdiction to another. A unitary state, or single-community, version of the model is also developed where an income tax is earmarked for redistribution and the property tax finances a public good.

A number of equilibrium conditions for both the unitary state and federation versions of the model are analytically established. Then numerically computed equilibria are developed in order to quantify the welfare and distributional effects of redistribution in both a unitary state and a federation. The numerically computed simulation model is also used to quantity the welfare and distributional effects of decentralization of public good provision. I show a paradoxical result in which poor households are better-off and rich households worse-off in a federal system mainly because majority voting leads to greater redistribution in a federation vis-à-vis a unitary state.
1. Introduction

This paper is concerned with the interrelationship between central government redistribution of income and the fiscal policies of local governments in an economic federation with two levels of government. I establish a two-stage voting political economy equilibrium in a model in which the central government’s (or state’s) income tax and redistribution grant is determined in the first-stage by majority vote of all residents. Then property tax rates and public good provision levels are determined in the second-stage by majority vote in each individual local municipality within the state. That is, the central government acts as the Stackelberg leader, and it is assumed the voters have perfect foresight on how the central government’s redistribution policy affects the political economy equilibrium fiscal policies at the local level. Voters understand that changing the central government’s tax/transfer policy will impact the allocation of households across the local communities and the concomitant property tax rates, public good provision, and prices of housing in each of the local communities.

I examine two versions of the model; a single-community and a multiple local communities version. In the single community model the population is fixed and the government levies an income tax that is used to finance redistribution and then imposes a property tax on housing to finance a public good. This case is similar to a situation in which a property tax is earmarked for particular publicly provided good, such as a school district using a property tax to finance public education expenses, and a coterminous municipal government imposing an income tax to contribute to the financing of other “reistributive” types of expenditures. In fact, most often states authorize the use of
income taxes only for local municipalities and not local school districts. Only three states permit school districts to use income taxes.¹

I formally prove the existence of endogenous voting equilibria in the first- and second-stage of voting for the single-community model when voters have the ability to perfectly predict the second-stage voting equilibrium given the first-stage voting outcome. Since many states that authorize the use of income taxes for local municipalities impose permissible ranges and maximum rates², I also examine how exogenous changes in income tax financed redistribution affect the voting on local public good provision. In this single-community case I show analytically that exogenous increases in income tax financed redistribution will cause the pivotal voter to prefer a greater amount of property tax financed public good if income taxes are nondistortionary. However, if there is a deadweight loss associated with income taxes, the exact relationship between the level of redistribution and public good provision is indeterminate. With distortionary income taxes, the relationship between redistribution and public good provision is ambiguous because of the following potential countervailing effects: 1) increasing redistribution increases the marginal cost of public good provision, 2) increasing redistribution changes the price per unit of housing for a given level of public good provision, creating opposing substitution and income effects on the demand for public good provision, and 3) increasing redistribution has an ambiguous directional effect on the pivotal voter’s after tax/transfer income.

Since in general the relationship between redistribution and public good provision is ambiguous, I develop a computational model in which parameters are calibrated to

²Ibid
realistic values to gain further insight into this relationship. In this single-community computational model, increases in the level of income tax financed redistribution leads to lower public good provision. I also use this calibrated computational model to show that the greater a household’s income, the lower the preferred income tax rate, given households know the median income household is pivotal in the second stage when voting over public good provision.

In order to quantify the welfare and distributional effects of redistribution policy for the single community case, I compare the computational two-stage voting political economy equilibrium to the computational equilibrium without redistribution. This comparison indicates that a majority of households prefer the equilibrium with income taxed financed redistribution, but there is an aggregate welfare loss.

In the multi-community version of the model households do not assume populations in their communities are fixed when voting on the property tax rate and public good expenditure in each of their communities. It is assumed households are able to predict how changes in their community’s tax/expenditure policy affect the tax base, population, and housing price in their community given that the housing prices and tax/expenditure policies in all other communities are fixed. In equilibrium, housing prices and local fiscal policies vary across communities, and every household locates in the community it most prefers. Even though a community’s population and tax base are not assumed to be fixed by households when voting within the community, existence of the second-stage voting equilibrium in each community is established just as in the single-community model.
However, proving the existence of a first-stage voting equilibrium in the multi-community version of the model is much more problematic than in the single-community case. This is because voters know how the central government’s income tax/transfer policy impact gross housing prices and public good provisions in all the local communities. In the single-community case there is only one housing price and public good provision associated with each possible central government redistribution policy. Hence, even though voters are voting over a multi-dimensional policy space, the structure of preferences makes analytically proving the existence of the first-stage voting equilibrium tractable. Given a fiscal policy outcome, variation in utility across households is due only to the differences in household income. In contrast, in the multi-community case housing prices and public good provisions vary across communities for every central government fiscal policy. Therefore, variation in utility across households is not only due to differences in income, but also due to the differences in local fiscal policies. Households are freely mobile and the different communities provide different levels of utility for each household for each possible central government fiscal policy. Consequently, it is not possible to prove a guaranteed existence of a first-stage voting equilibrium for the general form of the utility function.

Even though existence of the two-stage multi-community equilibrium is not guaranteed, I show computationally it is not unusual. Based on a realistically calibrated five community computational model, the household with the median endowed income is pivotal on the central government’s tax/transfer policy in pair-wise majority voting. Again, I show the first-stage voting equilibrium exists in this computational model even
though populations, housing prices, and the tax/expenditure policies in each of the local communities vary with changes in the central government’s fiscal policy.

I also show in the multi-community computational model centralized redistribution increases the amount of public good provision in almost all of the local communities, in contrast to the single-community model result in which redistribution reduces public good provision. Similar to the single community case, though, centralized redistribution causes an aggregate welfare loss even though a majority of households are made better-off.

Although the multi-community computational model uses parameter values that are calibrated based on the single-community computational equilibrium and the same household is pivotal in both equilibria, the multi-community equilibrium is characterized by a significantly higher level of redistribution. This result is due mainly to the fact that in the multi-community equilibrium the pivotal household resides in a local community in which aggregate community income increases with redistribution while overall state aggregate income decreases. Because of the higher level of centralized redistribution, poor households are better off and rich households are worse off in a multi-community system compared to a single community system. This result contradicts the generally held view in public finance that the poor are made worse-off by decentralization and the rich are made better-off.

Other researchers have investigated the choice of tax instruments in models with hierarchal governments. Aronsson and Blomquist (2008) considered a model in which a central government redistributes via a nonlinear income tax and a lump-sum transfer to each local government, while the local governments use proportional income taxes and
provide local public goods. They solve for the optimal tax/expenditure policies first for a unitary state where all decisions are made by the central government and then for decentralized system where the central government is the first mover. Nechyba (1997a) introduced myopic voting over local property tax rates, but a sophisticated planner still sets the state income tax rate.

Nechyba (1997b) also developed a model where majority voting determines the level of public expenditure at both government levels. This model consists of local public goods financed by property taxes and a national public good financed by a proportional income tax. Agents vote on each issue separately holding current public good levels (other than the ones being voted on) fixed. In this paper, Nechyba’s emphasizes the distinction made by Shepsle (1979) between “structurally” induced and “preference” induced equilibria. Structurally induced equilibrium results whenever structure is imposed on the model to produce equilibrium when a pure preference induced equilibrium does not exit. In Nechyba’s model, the imposed structure is (1) agents consider national and local public policy separately; (2) the state median pivotal voter holds as fixed all prices, everyone's location, and the majority rule local public good levels in all the communities, and; (3) the pivotal voter on the public good provision in each community takes as fixed the expenditure levels, tax rates, populations, and total housing demand in all other communities, and the population and total housing demand in his or her own community. This structure implies that they voters do not calculate the election's general equilibrium price and migration effects of changes in either state or local fiscal policy.
My model varies from Nechyba (1997b) in substantive ways. In my model the different levels of government have different fiscal policy roles. The higher level government’s role is redistribution and the local communities provide public goods or services. An even more significant difference is that voters are more sophisticated at both the state and local levels. As stated above, at the state level voters do not consider state policy independent of local fiscal policies. Voters are able to predict the general equilibrium effects of changes in state fiscal policy on housing prices, tax rates, and public good provision in the local communities. At the local level households are assumed to be “utility-takers,” which means when voting on their local fiscal policy households take as given the utility they would receive by migrating to any other community because gross housing prices and local public provision in all the other communities are assumed to be fixed. Households, however, are sophisticated enough to take into account how changes in their respective community’s tax rate and public good provision impact the community’s tax base through changes in current residents’ housing demands and the potential in- or out-migration of other households. As mentioned above, in Nechyba (1997b), households assume a fixed tax base when voting on their local community’s property tax rate.

Section 2 below presents the theoretical model. In section 2.1 the local communities model is described. In section 2.2 the state or the central government’s role in redistributing income and the associated impact on aggregate income is characterized. Households’ preferences and choices are formalized in section 2.3. The interrelationship between the two levels of government through the two stage voting process is described in detail in section 2.4.
In Section 3 the single community equilibrium conditions are analytically established and the quantitative analysis for this model is presented. I calibrate a computational model and examine household preferences over redistribution policy and quantify the welfare and distributional effects of redistribution.

The multi-community equilibrium conditions are established in Section 4. I computationally prove the first-stage voting equilibrium and analyze the welfare and distributional effects of redistribution. In Section 5 I contrast the equilibrium levels of redistribution between the single community and the multi-community fiscal structures. In this section I show the counterintuitive results discussed above that with centralized redistribution, a federal system with multiple local communities providing public goods will lead to a higher level of redistribution than a unitary state, and thus poor households prefer a federal system over a unitary state. Also, it is shown centralized redistribution in a federation generally leads to higher levels of local public good provision, while in a unitary system redistribution causes public good provision to fall.

Section 6 consists of a summary, conclusion, and descriptions of potential future research.

2. Model and Properties

The economy of the model consists of a continuum of households that differ only in their endowed income $y$.\(^3\) The distribution of income is represented by the continuous p.d.f., $f(y)$, normalized to unity. A household with endowed income $y$ will be referred to as household $y$. Mean and aggregate endowed income is denoted as $\bar{y}$, median endowed

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\(^3\) Assuming a single dimension of heterogeneity is restrictive. However, this simplification yields major benefits from the perspective of tractability while providing valuable insights.
income as \( y \). Consistent with most empirically observed income distributions, \( y > \bar{y} \).

All households have the same preferences represented by utility function \( U(g,h,b) \),

where \( g \) is expenditures on a locally publicly provided private good (e.g., per student educational expenditure), \( h \) is units of housing, and \( b \) is consumption of a numeraire bundle.

There is a two level hierarchical government structure. For example, a structure in which there are \( J \) number of local communities within the geographical boundaries of a central government or state. Each local government imposes an ad valorem property tax, \( t \), on the value of housing to finance \( g \). The higher level of government (state) imposes a proportional tax, \( m \), on income and redistributes the total tax receipts as an equal lump-sum grant, \( r \), to each resident household in the state. In the following subsections, I formally describe the local communities model, the state level model, household preferences and choices, and lastly, the interactions and relationships between the two levels of government.

2.1 Local Communities Model

The \( J \) communities in the state may differ in their land areas as a proportion of total homogeneous land area in the state, \( L_{ij}, j = 1,2,\ldots,J \). The default range of the index \( j \) (and \( i \)) is from 1 to \( J \). Each local government imposes an ad valorem property tax, \( t^j \), on the value of housing to finance \( g^j \) and balances its budget. Housing in each community is produced by price-taking firms from land and non-land factors via a constant-returns neoclassical production function. I assume absentee housing owners who supply housing competitively because it is most standard and simplest. The price of non-land factors is
assumed fixed and uniform throughout the state. The housing supply function in community \( j \), \( C^j \), is represented by:

\[
H_s^j(p_h^j) \equiv L_j h_s(p_h^j) 
\]

(1)

where \( p_h^j \) is the supplier price of housing in \( C^j \), and \( h_s(p_h^j) \) is housing per unit of land in \( C^j \). The price of housing in each community is determined by a competitive market equilibrium in which:

\[
H_s^j(p_h^j) = H_d^j(p_h^j(1+t^j)) 
\]

(2)

where \( H_d^j(p_h^j(1+t^j)) \) is housing demand in \( C^j \).

Households in the state are able to migrate from one community to another and locate in the jurisdiction they most prefer. The measure of households with income \( y \) that locate in \( C^j \) is denoted by:

\[
f_j(y) \text{ with support } S_j \subset S \equiv [y_{\text{min}}, y_{\text{max}}] 
\]

(3)

where \( S \) is the support of the income distribution in the state. The government budget constraint for \( C^j \) is:

\[
g^j \int_{y_{\text{min}}}^{y_{\text{max}}} f_j(y) dy = t^j p_h^j H_s^j(p_h^j) 
\]

(4)

The expenditure/tax policy in community \( j \), \( (g^j, t^j) \), is chosen by a majority-rule vote among the households living in community \( j \). Within \( C^j \) the set of all \( (g, t) \) combinations voters can vote over is defined as the local Budget Possibility Frontier (BPF). The BPF in each community is determined by the housing market equilibrium and the balanced government budget. Since \( t^j \) is an ad valorem tax on housing, the
The gross-of-tax price per unit of housing in $C^j$ is $p^j = p^j_h(1 + t^j)$. The indirect utility of a household $y$ that locates in $C^j$ is a function of $(g^j, p^j)$. Hence, when voting within a community on a $(g, t)$ pair, voters’ preferences are based on the $(g, p)$ pair associated with each possible $(g, t)$ combination. Therefore, the $BPF$ within each community will subsequently be defined as the set of all possible $(g, p)$ combinations.

The complete characterization of $BPF$ depends on voters’ perceptions on how the private-market equilibrium in the community will be affected by the community’s public policy choices. Which $(g, p)$ combinations voters perceive are possible in their community depend on how voters perceive the relationship between local public policies and the population and the tax base in their community. There are many possible ways to characterize the $BPF$, depending on the degree of voter sophistication in anticipating the consequences of policy changes within a community. My characterization of voting behavior draws on modern club theory and assumes that individuals are utility-takers.$^4$ This means voters assume that the tax/expenditure policies $(t, g)$ and housing prices in all the other communities are fixed. Employing this utility-taking assumption, voters predict how the private market equilibrium would change in response to a prospective local policy change. For example, a voter assumes the price of housing in his/her community is affected by changes in his/her local government’s budget through both changes in housing demand by current residents and migration into or out of the community, taking as given the state’s tax/transfer policy and policies and prices in other communities.

$^4$The theory of clubs was initiated by Buchanan (1965). See also Ellickson (1973, 1979), Scotchmer and Wooders (1987), Cornes and Sandler (1996), Gilles and Scotchmer (1997), and Ellickson, Grodal, Scotchmer, and Zame (1999).
Hence, the utility-taking specification is consistent with the following timing of choices. First, households choose an initial jurisdiction where they will vote. Second, they vote in their jurisdiction over a property tax taking as given the *equilibrium* utility levels obtainable in all jurisdictions *other than their own*, anticipating in- and out-migration to and from their own jurisdiction whenever such would provide higher utility. Lastly, housing market clearance and local government budget balance are satisfied given the property tax, and jurisdictional choices are utility maximizing given all equilibrium values. In the utility-taking equilibrium studied, the initial residence choices correspond to the final residence choices, consistent with equilibrium since households anticipate equilibrium values. While no one actually migrates in equilibrium, the possibility of moving between jurisdictions has substantial effects on equilibrium.5

### 2.2 State Level Model

The higher level of government (state) imposes a proportional tax, \( m \), on income and redistributes the total tax receipts as an equal lump-sum grant, \( r \), to each resident household in the state. The tax/expenditure policy \((m, r)\) is determined by majority rule voting over all the residents within the state.

Since in my model endowed income is exogenous and there is no labor market, the deadweight loss due to labor market distortions that can potentially be created by a state income tax, \( m \), is not directly captured. Also not necessarily captured is the

5 The utility-taking equilibrium does not correspond to the more appealing subgame-perfect Nash equilibrium. In the utility-taking equilibrium, voters anticipate all the effects of migration on their own jurisdiction, but hold constant utilities, not just property taxes, in other jurisdictions. In the related Nash equilibrium, voters would need to anticipate the effects of moving across jurisdictions on all equilibrium values in all jurisdictions (holding constant property tax rates). Computing the Nash equilibrium would be *very* difficult in a five jurisdiction model. Hopefully, the simpler utility-taking alternative is not a bad approximation.
deadweight loss caused by the effect of income taxes on tax avoidance through the substitution of tax-exempt compensation (e.g. employer-paid health insurance) for non-exempt compensation and through the substitution of fully or partially deductible consumption (e.g. education, donations, owner-occupied housing) for non-deductible consumption. Feldstein (1999) shows that deadweight loss of tax avoidance through changes in the form of compensation and through changes in the patterns of consumption can be evaluated as the deadweight loss of an excise tax on non-deductible consumption.

In order to include any potential deadweight loss associated with income taxes, the after state tax/transfer income of household $y$ is calculated as $y^*$:

$$y^* = y(1-(1+\gamma)m)+r$$

(5)

where $\gamma$ is a deadweight loss factor associated with $m$ and $\gamma > 0$.

It can be shown that given $\gamma$ and $m$, the effective before state income tax income, $x$, of a household with endowed income $y$ is:

$$x = \frac{y(1-(1+\gamma)m)}{1-m}.$$  

(6)

It is assumed that $(1+\gamma)m < 1$ so that aggregate before tax income $\geq 0$. Based on equations (5) and (6):

$$y^* = x(1-m) + r = y(1-(1+\gamma)m)+r$$

(7)

The state government’s budget constraint is:

$$r = m\int_0^\infty \frac{y(1-(1+\gamma)m)}{1-m} f(y)dy = \frac{m(1-(1+\gamma)m)}{(1-m)} \bar{y}.$$  

(8)

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6The derivation of equation (6) is available on request.
All the possible \((m,r)\) combinations state residents can vote over must satisfy (8)

I define this set of all possible \((m,r)\) combinations as the state’s *Redistribution Possibility Frontier (RPF)*.

### 2.3 Households

Households in the state are able to migrate from one community to another and locate in the jurisdiction they most prefer. A household locates in the community with the tax/expenditure policy for which the household obtains the highest utility. I adopt the following functional form for utility:

\[
U(g, h, b) = f(g)[u(h, b) + \phi]
\]  \hspace{1cm} (9)

I assume \(g\) is separable in a household’s utility from housing and numeraire consumption. This simplifying assumption implies that a household’s demand for housing and the numeraire good are not directly dependent on the amount of \(g\) provided in a community. These demands are indirectly dependent on the expenditure on \(g\) through the taxes used to finance \(g\). I also assume that \(u(h, b)\) is homogeneous of degree 1. This assumption is consistent with the empirical evidence on housing demand (Harmon, 1988), which suggests that the income elasticity of housing demand is approximately one. This homogeneity assumption is widely used in the optimal taxation literature, in dynamic macroeconomic simulation models, and in a variety of other applications in economics.

From equation (5) the budget constraint of a household with income \(y\) that locates in a particular community \(j\) is as follows:

\[
y(1-(1+\gamma)m) + r = p'h + b
\]  \hspace{1cm} (10)
Since $u(h,b)$ is homogeneous of degree 1, household $y$'s housing demand in $C^j$ is linear in income and can be represented by $(y(1-(1+\gamma)m)+r)h_y(p^j)$. Hence, the housing market equilibrium condition for $C^j$ (equation (2)) can be written:

$$h_y(p^j)\int_{y_{\text{min}}}^{y_{\text{max}}} (y(1-(1+\gamma)m)+r)f_j(y)dy = L_jh_y(p^j) \quad (11)$$

Linear homogeneity of $u(h,b)$ also implies that the corresponding indirect utility function is linear in income, a property that will prove central in my analysis of voting equilibrium:

$$V(g,p,m,r;y) = f(g)[(y(1-(1+\gamma)m)+r)w(p) + \phi] \quad (12)$$

Without loss of generality, it is assumed that $V(p,g;y)$ is twice continuously differentiable and strictly quasi-concave in $(g,p)$ for $(g,p) > 0$ and that the Inada condition $V_g \to \infty$ as $g \to 0$ holds. It is also assumed that $f(0) \geq 0$ and $\phi < 0$ so that the preference function satisfies the usual single crossing condition that the slope of indifference curves in the $(g,p)$ plane increases with income.

### 2.4 Two-Stage Sequential Voting and the Hierarchical Intergovernmental Fiscal Relationship.

As described above, the tax/expenditure polices of each level of government is determined by majority rule voting over all the residents living within the respective government’s jurisdiction. It is assumed the tax/expenditures policies of the different levels of government are determined sequentially. The higher level government acts as the Stackelberg leader with perfect foresight, whereas the local governments act as followers. In other words, the tax/transfer policy $(m,r)$ of the higher level government is
determined first by a vote over all residents of the state. Then, given the state’s tax/transfer policy, the tax and expenditure policy \((g, p)\) in each local community is determined by a vote by only the residents of each respective community. This assumption is in line with most earlier studies on optimal taxation and public expenditures in economic federations (Aronsson and Bloomquist 2008).

In the first stage of voting, the state’s \((m, r)\) pair is chosen from all the combinations on the states \(RPF\) by a majority-rule vote of all the residents of the state. The total population within the state is fixed.\(^7\) A voter’s preferences over points on the \(RPF\) depends not only on his/her net tax payments to the state for each \((m, r)\) combination, but also on the relationship between state redistribution policy and its effects on equilibrium tax rates, public good expenditures, and housing prices in his/her chosen community. It is assumed voters can perfectly predict how changes in state redistribution policy affects the allocation of households among the local communities and the corresponding equilibrium tax rates, public good expenditures, and housing prices in all the communities given that voters are “utility takers” when voting on local budget policy.

In the second stage of voting, the \((g, p)\) pair in each local community is chosen from all the combinations on the community’s \(BPF\) by a majority-rule vote among the residents of the community. The \(BPF\) in each community not only depends on the utility-taking behavior of households, but also on the \((m, r)\) combination chosen at the state level. Each \((m, r)\) on the \(RPF\) will be associated with different \(BPF\)’s at the local level.

\(^7\)This assumption is consistent with empirical finding by Leigh (2008). He finds on aggregate more redistributive state taxes do not substantially affect interstate migration.
Therefore, the $BPF$ within each community will subsequently be defined as the set of all possible $(g, p)$ combinations given a state policy $(m, r)$.

In the model as presented heretofore a higher level of government (eg. state) imposes a redistributive income tax on households and multiple lower level governments impose an ad valorem property taxes to finance public good expenditures within each community. There are different pivotal voters for the state income tax and each community’s property tax. Each community’s tax/expenditure policy is voted after the higher level government’s tax/transfer policy is determined. However, as mentioned in the introduction, there is also a single-community interpretation of the model in which a single government imposes a property tax on housing to finance a public good and levies an income tax that is used to finance redistribution. In the following section I examine and establish necessary conditions for the existence of an equilibrium in the single-community case. In later sections I extend the model and examine equilibrium conditions with multiple local communities that levy property taxes to finance local public goods and a higher level of government that levies a redistribute progressive income tax on all households across all the local communities.

3. Single-Community Equilibrium Conditions

In this single community model there are no migration affects on the local $BPF$, so the utility-taking specification for voting at the local level does not apply. The single-community $BPF$ still reflects the ability of households to perfectly anticipate how changes in the local government's budget affects housing prices through changes in housing demand. Hence, the timing of choices and voting are as follows. First, the
households vote over the \((m, r)\) pairs on the \(RPF\). Households then vote over the \((g, p)\) pairs on the \(BPF\) given the chosen \((m^*, r^*)\). Lastly, housing market clearance and local government budget balance are satisfied given \((g^*, p^*)\) and \((m^*, r^*)\).

Housing supply in the single-community case is the aggregation of the jurisdictional housing supplies of the multi-community case. Thus, equilibrium in the single government model is defined by the following conditions:

1. Aggregate demand for housing equals supply of housing.

\[
\int_{0}^{\infty} (y(1-(1+\gamma)m) + r)h_d(p) = h_f(p_h) \tag{13}
\]

2. The government’s two budget constraints hold.

\[
r = \frac{m(1-(1+\gamma)m)}{(1-m)}\bar{y}, \tag{14}
\]

\[
g = tp_hh_d(p)(\bar{y}(1-(1+\gamma)m) + r) \tag{15}
\]

3. There is a majority-rule voting equilibrium on the government’s tax and expenditure policies.

The \(RPF\) is equation (14) and the \(BPF\) for any \((m, r)\) combination is defined by equations (13) and (15). The third condition requires that a sequential majority voting equilibrium exists. That is there exists a first-stage voting equilibrium in which preference over the possible \((m, r)\) combinations on the \(RPF\) depend on how redistribution policy affects the choice of \((g, p)\) in the second stage, and a second-stage voting equilibrium over the \((g, p)\) bundles on \(BPF\) given the redistribution policy \((m, r)\) .
3.1 Establishing the first-stage voting equilibrium for the Single Community Model

Individual households’ preferences over the RPF in the first-stage of voting will partially depend on how redistribution policy affects the median voter’s choice of \((g, p)\) in the second stage. Recall voters have perfect foresight about how the values of \((g, p)\) determined in the second stage of voting depend on the \((m, r)\) pair determined in the first stage of voting. Proposition 1 proves the existence of a MVE on the RPF in the first stage of voting given that the \((g, p)\) pair determined in the second stage is a function of \((m, r)\).

**Proposition 1:** The Majority Voting Equilibrium on the Redistribution Possibility Frontier is the \((m, r)\) point most preferred by the community’s median endowed income voter.

**Proof:** Let \(\tilde{x} = (\tilde{m}, \tilde{r}, g(\tilde{m}, \tilde{r}), p(\tilde{m}, \tilde{r}))\) denote the point most preferred on the RPF by the median endowed income voter, \(y\). To form a contradiction suppose there exists a point \(x = (m, r, g(m, r), p(m, r))\) that defeats \(\tilde{x}\). Let \(\Delta V(y)\) be the difference in utility that voter \(y\) obtains between \(\tilde{x}\) and \(x\):

\[
\Delta V(y) = f\left( g(\tilde{m}, \tilde{r}) \right) \left( \gamma (1 - (1 + \gamma)\tilde{m}) + \tilde{r} \right) w(\tilde{p}(\tilde{m}, \tilde{r})) + \phi \\
- f\left( g(m, r) \right) \left( \gamma (1 - (1 + \gamma)m) + r \right) w(p(m, r)) + \phi
\]

(16)

It cannot be the case that \(\Delta V(y) < 0\) for all \(y\). This would contradict the assumption that \(\tilde{x}\) is the point most preferred by the median endowed income voter. Alternatively, if \(\Delta V(y) > 0\) for all \(y\), then \(\tilde{x}\) is unanimously preferred to \(x\). If all voters

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8The strategy of proof of this proposition is due to Cassidy (1990) who exploits the linearity of the indirect utility function in income to study voting equilibrium in a model with a flat grant financed by a property tax.
are indifferent between \( \tilde{x} \) and \( x \) (i.e., \( \Delta V(y) = 0 \) for all \( y \)), then we adopt the convention that \( \tilde{x} \) is chosen.

This leaves cases where some voters strictly prefer \( \tilde{x} \) to \( x \) while others strictly prefer \( x \) to \( \tilde{x} \). For these cases, the linearity of \( \Delta V(y) \) implies that there is a unique \( \hat{y} \) such that \( \Delta V(\hat{y}) = 0 \). There are two possibilities. One is that voters preferring \( x \) to \( \tilde{x} \) have incomes less than \( \hat{y} \). If \( x \) defeats \( \tilde{x} \), these voters comprise more than half the population. This implies \( \tilde{y} < \hat{y} \) which in turn implies that \( \tilde{y} \) prefers \( x \) to \( \tilde{x} \). This is a contradiction since \( \tilde{x} \) is \( \tilde{y} \)'s most preferred outcome. The other alternative is that voters with income greater than \( \hat{y} \) prefer \( x \) to \( \tilde{x} \). Since \( x \) defeats \( \tilde{x} \), these voters comprise more than half the population. This implies \( \tilde{y} > \hat{y} \) which implies that \( \tilde{y} \) prefers \( x \) to \( \tilde{x} \). This is again a contradiction since \( \tilde{y} \) prefers \( \tilde{x} \) to \( x \).\footnote{Since the median endowed income voter is decisive in both stages of voting and voters have perfect foresight, the outcome of the sequential process in which the \((m, r)\) pair is voted on in the first stage and \((t, g)\) pair is voted on in the second stage is equivalent to the outcome of voting simultaneous over all four policy variables. This equivalency voting result only holds for the single-community case and not the multi-community case presented later in this paper.}

Proposition 1 establishes the existence of voting equilibrium in the first-stage of voting under the condition that voters know what the second-stage voting equilibrium will be given the redistribution policy chosen in the first-stage. In the next section, the existence of the second stage voting equilibrium given any \((m, r)\) pair is established, as well as a characterization of the effects redistribution has on the second-stage equilibrium \((g, p)\) values.
3.2 Characterization of the second-stage voting equilibrium for the Single-Community Model

As shown in equation (12), indirect utility can be represented by:

\[ V(y) = f(g)((y(1-(1+\gamma)m)+r)w(p)+\phi) \]

Hence, given \((m,r)\) utility for any household \(y\) is a function of variables \(\phi\) and \(\gamma\). Since \(f(0) \geq 0\), \(\phi < 0\) and \((1+\gamma)m < 1\), my preference function satisfies the single crossing condition that the slope of indifference curves through any point in the \((g,p)\) plane increases with income:

\[
\frac{\partial^2 p}{\partial g \partial y} \bigg|_{V_{\text{const.}}} = \phi \frac{df(g)}{dg} \frac{f(g)(1-(1+\gamma)m)}{dw(p)} \frac{dw(p)}{dp} < 0 \quad \text{(17)}
\]

because \(\frac{df(g)}{dg} > 0\) and \(\frac{dw(p)}{dp} < 0\).

It is a well known result in the literature (Roberts 1977, Westoff 1977, Epple and Romer 1991, Epple and Platt 1998, Meltzer and Richard 1981) that if preference exhibit the single crossing property, the median-\(y\) household’s (\(\tilde{y}\)’s) most preferred point on the BPF is the majority voting equilibrium. The \(\tilde{y}\)’s choice of public good provision will depend on the income tax financed level of redistribution. It is assumed in this model that the level of redistribution is endogenously determined by majority voting. However, in some states local governments are required to have authorization from the state legislature to impose an income tax. That authorization usually includes a permissible
range of rates.\textsuperscript{10} Hence, it is of interest to investigate how exogenous changes in income tax financed redistribution impacts $\bar{y}$’s choice of public good provision in this single community model.

If redistribution has little or no deadweight loss associated with it, i.e. $\gamma = 0$, then changing the redistribution policy $(m, r)$ will have little or no affect on the $BPF$ because housing demand is linear in income and thus the income elasticity of demand for housing $=1$. Thus, if $\gamma = 0$, changes in $(m, r)$ will only affect the level of public good provision through the affects on the $\bar{y}$’s preferences over $(g, p)$. Proposition 2 below shows that increases in level of redistribution, $(m, r)$, will result in greater public good provision, higher housing prices, and a higher property tax in the community if redistribution has little or no deadweight loss.

**Proposition 2:** Increasing redistribution results in higher public good provision $(g)$, property tax rate $(t)$ and housing price $(p)$ if $\gamma = 0$.

**Proof:**
If $\gamma = 0$ then the $RPF$ represented by equation (14) collapses to:

$$r = m\bar{y}$$

(18)

The slope of $\bar{y}$’s indifference curve in the $(g, p)$ plane increases with $m$:

$$\frac{\partial^2 p}{\partial g \partial m} \bigg|_{v=\text{const.}} = \frac{\phi \frac{df(g)}{dg} f(g)(\bar{y} - \bar{y}) \frac{dw(p)}{dp}}{\left(f(g)(y(1-m) + m\bar{y}) \frac{dw(p)}{dp}\right)^2} > 0 \tag{19}$$

Therefore, as \( m \) increases, the slope of the pivotal voter’s (\( \bar{y} \)) indifference curve increases and the \( BPF \) (thus the possible \((g, p)\) combinations) does not change.

Illustrated in Figure 1 is \( \bar{y} \)'s indifference curve, \( \tilde{V}(m^1) \), through the point \((g^1, p^1)\) on the \( BPF \) that maximizes \( \bar{y} \)'s utility when \( m = m^1 \). Also shown is \( \bar{y} \)'s indifference curve, \( \tilde{V}(m^2) \), through \((g^1, p^1)\) where \( m = m^2 > m^1 \). Suppose \((g^2, p^2)\) is \( \bar{y} \) most preferred point on the \( BPF \) if \( m = m^2 \). Point \((g^2, p^2)\) cannot be in Region A (all the points to the right of \( \tilde{V}(m^1) \)) because no points on the \( BPF \) could be in this region otherwise \((g^1, p^1)\) could not maximize \( \bar{y} \)'s utility given \( m^1 \). Also, \((g^2, p^2)\) cannot be in region B (all the points to the left of the dashed vertical line and \( \tilde{V}(m^1) \)) because \( \bar{y} \) prefers \((g^1, p^1)\) to any point in this region. Hence, point \((g^2, p^2)\) must be in region C (all the points to the right of the dashed line and left of \( \tilde{V}(m^1) \)). Region C contains only points \((g, p)\) where \( g > g^1 \) and \( p > p^1 \). Since the income elasticity of demand = 1, increases in \((m, r)\) can only result in higher \( g \) if \( t \) increases.\(^{11} \)

Proposition 2 holds because if \( \gamma = 0 \), increasing the level of redistribution by increasing \( m \) does not affect the tax base and thus the \( BPF \), but it does increase the after tax/transfer income of the median voter. The single crossing property of indifference curves implies a household’s preferred \( g, p, \) and \( t \) on the \( BPF \) all increase with income. If \( \gamma > 0 \), increasing \( m \) has an ambiguous effect on the amount of public good provision that the median voter prefers in the second stage of voting. One cause of this ambiguity is due to the effect increasing \( m \) has on the shape of the \( BPF \). The other cause of this

\(^{11} \) If the \( BPF \) is not differentiable at \((g^1, p^1)\), then \((g^1, p^1) = (g^2, p^2)\).
FIGURE 1
ambiguity concerns how an increase in $m$ impacts $\tilde{y}$'s after income tax/transfer income.

In order to address the first effect, consider first the general equation for the slope of the $BPF$ given $m^{12}$:

$$\frac{\partial p}{\partial g} = \frac{p_h \frac{\partial t}{\partial g}}{1 - \frac{\varepsilon_d}{\varepsilon_s}}$$

where $\varepsilon_d$ and $\varepsilon_s$ are the price elasticity of demand and supply for housing respectively.

Since the median voter would only choose a point on the $BBF$ where $\frac{\partial t}{\partial g} > 0$, the slope on the relevant parts of the $BPF$ is positive. As an illustration of how changes in $m$ affect the $BPF$ and lead to ambiguity in the relationship between $m$ and $g$, I derive an explicit expression for (20) by assuming $u(h,b)$ in equation (9) is Cobb-Douglas such that $u(h,b) = h^\alpha b^{(1-\alpha)}$ where $0 < \alpha < 1$, and the housing supply function has a constant elasticity of supply form $H_s(p_h) = p_h^\theta$. In this example, this form of $u(h,b)$ is consistent with the assumption that utility is linearly homogeneous in $h$ and $b$. Also, $\varepsilon_d = -1$ and $\varepsilon_s = \theta$, and based on equations (13), (14), and (15), the slope of the $BPF$ for a given $m$, equation (20), is:

$$\frac{\partial p}{\partial g} = \frac{\theta \alpha \tilde{y}^*}{1+\theta} \left( \alpha \tilde{y}^* - g \right)^{-2\theta-1}$$

where $\tilde{y}^* = \frac{\tilde{y}(1-(1+\gamma)m)}{1-m}$.

Based on (21), the slope of the $BBF$ increases in $g$ and $m$, i.e., $\frac{\partial^2 p}{\partial^2 g} > 0$ and $\frac{\partial^2 p}{\partial g \partial m} > 0$.

---

12The derivation of equation (20) is available on request.
Figure 2 below illustrates for this example the impact increasing $m$ has on the $BPF$. Consider two different income tax rates such that $m^1 < m^2$. The $BPF$ for $m^i$ is represented by $BPF(m^i)$. Since $m^1 < m^2$, at $(g, t) = (0,0)$, the price of housing is less given $m^2$ because mean income is lower, thus housing demand is lower and $BPF(m^2) < BPF(m^1)$. However, because the slope of a $BPF$ increases in $m$ given $g$, $BBF(m^2)$ will eventually exceed $BPF(m^1)$.

As can be deduced from Figure 2, an increase in $m$ will impact $\tilde{y}$’s demand for $g$ (denoted by $\tilde{g}(m^i)$) even if $\tilde{y}$’s after tax/transfer income, $\tilde{y}^*$, does not change as $m$ changes. To see this, denote $\tilde{y}$’s optimal choices of $h$ and $b$ given a value for $g$ on $BPF(m^i)$ as $h(p(g;m^i), \tilde{y}^*)$ and $b(p(g;m^i), \tilde{y}^*)$, respectively, where function $p(g;m^i)$ is defined by $BPF(m^i)$. Household $\tilde{y}$’s total household cost or expenditures (which equals $\tilde{y}^*$) when optimizing given a value for $g$ on $BPF(m^i)$ is:

\[
Total\ Cost = p(g, m^i)h(p(g;m^i), \tilde{y}^*) + b(p(g;m^i), \tilde{y}^*). \tag{22}
\]

Thus, by the envelope theorem, $\tilde{y}$’s marginal cost of $g$ is:

\[
MC(g;m^i) = \frac{dp}{dg}(g;m^i)h(p(g;m^i), \tilde{y}^*) \tag{23}
\]

where $\frac{dp}{dg}(g;m^i)$ is the slope of $BPF(m^i)$. 

27
Figure 2

\[ m^1 < m^2 \]

\[ \text{BPF}(m^1) = p(g; m^1) \]

\[ \text{BPF}(m^2) = p(g; m^2) \]

\[ p(\hat{g}(m^1); m^1) \]

\[ p(\hat{g}(m^1); m^2) \]

Increasing \( t \)
Given $m^l$, $\bar{y}$’s demand for $g$ is represented as $\bar{g}(m^l)$ in Figure 2 and the corresponding price on the BPF($m^l$) is represented by $p(\bar{g}(m^l);m^l)$. The marginal cost of $\bar{g}(m^l)$ at $(\bar{g}(m^l), p(\bar{g}(m^l);m^l))$ is:

$$MC\left(\bar{g}(m^l), p(\bar{g}(m^l);m^l)\right) = \frac{dp}{dg} \left(\bar{g}(m^l); m^l\right) h(p(\bar{g}(m^l);m^l), \bar{y}^*) \quad (24)$$

If $m$ increases to $m_2$, the marginal cost of $\bar{g}(m^l)$ is:

$$MC\left(\bar{g}(m^l), p(\bar{g}(m^l);m^2)\right) = \frac{dp}{dg} \left(\bar{g}(m^l); m^2\right) h(p(\bar{g}(m^l);m^2), \bar{y}^*) \quad (25)$$

Since the slope of the BPF increases in $m$, i.e. $\frac{\partial^2 p}{\partial g \partial m} > 0$, and $p(\bar{g}(m^l);m^l) > p(\bar{g}(m^l);m^2)$, it must be the case $MC\left(\bar{g}(m^l), p(\bar{g}(m^l);m^2)\right) > MC\left(\bar{g}(m^l), p(\bar{g}(m^l);m^l)\right)$. That is, the marginal cost of $\bar{g}(m^l)$ increases as $m$ increases, reducing $\bar{y}$’s demand for $g$ if $\bar{y}$’s after tax/transfer income does not change. The marginal cost of the public good to the pivotal voter increases because an increase in $m$ reduces the tax base, thus a greater increase in $t$ is required to finance an additional infinitesimal unit of $g$, which leads to a greater additional increase in $p$ for each additional unit of $g$. However, since $p(\bar{g}(m^l);m^l) > p(\bar{g}(m^l);m^2)$, there are also a substitution and an income effect on $\bar{y}$’s demand for $g$ due to this decrease in housing price. Thus, if $m$ increases from $m^l$ to $m_2$, the increase in the marginal cost of $g$ at $\bar{g}(m^l)$, and the substitution effect of the decrease in $p$ from $p(\bar{g}(m^l);m^l)$ to $p(\bar{g}(m^l);m^2)$, reduces the median voter’s demand for public good provision, while the income effect of the decrease in $p$ increases the median voter’s
demand for public good provision. Hence, because of the effect increasing $m$ has on the shape of the $BPF$, if the median voter’s after tax/transfer income does not change, increasing $m$ has an ambiguous effect on equilibrium public good provision. If the median voter’s after tax/transfer income, $\bar{y}^*$, does change when $m$ changes, there is an additional ambiguity concerning how increases in $m$ affect $\bar{g}$. Given any $BPF$, the greater (lower) $\bar{y}^*$ is, the greater (lower) is $\bar{g}$. Whether $\bar{y}^*$ increases or decreases as $m$ changes depends on the parameter $\gamma$ and the magnitude of the inequality in endowed income measured by the difference between $\bar{y}$ and $\tilde{y}$.

The exact relationship between the $(m,r)$ pair chosen in the first stage of voting and the choice of $(g,p)$ in the second stage of voting depend on the income and price elasticities of housing demand and supply, the distribution of income, and the deadweight loss associated with income tax financed redistribution. Therefore, in the next section I calibrate and develop a computational model to further investigate this relationship between $(m,r)$ and $(g,p)$.

3.3 Single-Community Computational Model Equilibrium

In this computed equilibrium, just as I did in the illustrative example I used above to explain the shape of the $BPF$, I chose a utility function where $u(h,b)$ in equation (9) is Cobb-Douglas. The full utility function is assumed to be:

$$U(g,h,b) = g^\beta (h^a b^{1-a} + \phi)$$  \hspace{2cm} (26)

This function is consistent with the assumptions adopted above—that $g$ is separable in a household’s utility from housing and numeraire consumption and as well
as utility being linearly homogeneous in $h$ and $b$. The corresponding indirect utility function is

$$V(y) = g^\beta [\alpha^\alpha (1-\alpha)^{(1-\alpha)} (y(1-(1+\gamma)m)+r)p^{-\alpha} + \phi]$$  

(27)

Again, just as in the BPF example above, the following constant-elasticity of housing supply function is adopted:

$$H_s(p_h) = p_h^\theta$$  

(28)

The price elasticity of housing supply, $\theta$, is the ratio of nonland to land inputs in the production of housing. Based on available evidence regarding the share of land and non-land inputs in housing, this parameter is set equal to three. This supply function is implied by a constant returns to scale Cobb-Douglas production function.$^{13}$

The distribution of endowed income within the state is calibrated using data from the 1999 American Housing Survey (AHS).$^{14}$ Median income reported by the AHS is $36,942. Using data for the 14 income classes reported by the AHS, I estimate mean household income to be $54,710. These values and the assumption that the income distribution is lognormal imply $\ln y \sim N(10.52, 0.887)$.

Recall that the conditions for equilibrium are that the housing market clears, the government’s two budget constraints hold, and there is a majority-rule equilibrium determining tax and expenditure policy. The calibrated values of $\beta$, $\alpha$, $\phi$, and $\gamma$ are based on the following constraints:

---

$^{13}$ This housing supply elasticity is within the range of estimates for new housing, though estimates vary substantially. See Dipasquale (1999), Blackley (1999), and Somerville (1999). Dipasquale and Wheaton (1992) estimate the long run rental housing supply elasticity to be 6.8. Other estimates also find a higher elasticity than 3 (see Mayer and Somerville, 2000, and Epple, Gordon, and Sieg, 2007).

$^{14}$ http://www.census.gov/hhes/www/housing/ahs/99dtchrt/tab2-12.html
1) In the single jurisdictional equilibrium the pivotal voter chooses \( t = 35\% \) and \( m = 2\% \). A \( t = 35\% \) implies a tax rate on property value that is realistic, on the order of 2.5\% to 3.0\%.\(^{15}\) A \( m = 2\% \) is the approximate median and mode of local income tax rates reported in Henchman (2008).

2) I derive the demand functions for \( h \) and \( g \) assuming both are privately provided goods:

\[
h = \frac{\alpha}{(1 + \beta) p} \left[ y - \frac{\beta \phi}{\alpha^a (1 - \alpha)^{(1-a)} p^{-a}} \right]
\]

\[
g = \frac{\beta}{(1 + \beta)} \left[ y + \frac{\phi}{\alpha^a (1 - \alpha)^{(1-a)} p^{-a}} \right]
\]

The share of aggregate income spent on housing is set equal to 1/3, which is in the range of values estimated in the literature (see Hanushek and Quigley (1980)). The share of aggregate income spent on \( g \) is set equal to 9\%, which is approximately the share of GDP spent on local public goods\(^{16}\).

---

\(^{15}\) Observed property tax rates are expressed as a percent of property value. In our model, rates are expressed as a percentage of annual implicit rent. Employing the approach of Poterba (1992), Calabrese and Epple (2006) conclude that tax rates on annualized implicit rents can be converted to rates on property values using a conversion rate on the order of 7\% to 9\%. Thus, our annualized rate of .35 translates to a tax rate on property value on the order of 2.5\% to 3\%, which is the order of magnitude of observed property tax rates.

\(^{16}\) Data for this approximation are from the 2008 Statistical Abstract Tables 442 and 645 for 2004.
The calibrated parameter values of the utility function are \( \beta = 0.107, \alpha = 0.366, \phi = -711, \) and \( \gamma = 0.45 \textsuperscript{17} \). Given these calibrated parameter values, Table 1 presents how changes in the community’s redistribution policy \((m, r)\) affects the median voter’s choice of triple \((\tilde{p}, \tilde{g}, \tilde{t})\). The maximum value of 

\[ m = \frac{1}{1+\gamma} = \frac{1}{1.45} \approx 0.689. \]

Increasing the income tax causes equilibrium housing price and public good provision to fall for all possible values of \( m \). This result is opposite of the effect of increases in \( m \) on \((g, p)\) when there is little or no deadweight loss associated with \( m \), i.e., \( \gamma \approx 0 \). In addition, the lump-sum grant increases in \( m \) up to \( m = 44.2\% \), at which point \( r \) is maximized, \( r = $15,538 \).

\textsuperscript{17} It can be shown that the value of \( \gamma \) can be reasonably approximated by the elasticity of reported or taxable income with respect to the income tax rate. (The authors upon request will provide the derivation of this approximation.) Recent estimates for the value of \( \gamma \) for federal income tax are within the approximate range of \((0.5, 1)\) (Feldstein (1999) pgs 676-677). The boundaries for the plausible range of values of \( \gamma \) for a local income tax are most likely lower than the boundaries for federal income tax. This is because local governments generally do not permit as many exemptions and deductions as the federal government. The calibrated value of 0.45 is below the lower bound.
Table 1

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</table>

* MVE rate of $m = 2\%$.
** Maximum value of lump-sum redistribution $r = 15,538$, $m = 44.2\%$.
*** Maximum possible value of $m = \frac{1}{1 + \gamma} = 68.9\%$.
3.4 Examination of household preference over Redistribution Policy in the Single Community Case.

This calibrated computation model can also be applied to show that all \( y > (\prec) \tilde{y} \) prefer a lower (higher) income tax rate than the equilibrium \( \tilde{m} \) when voting on the redistribution policy in the first-stage, given that voters know that the median voter is in the second stage. This result implies prefer \( g > (\prec) \tilde{g} \) and \( p > (\prec) \tilde{p} \). To see this consider household preferences over feasible policies in the \( (m, g) \) space. The slope of a household \( y \)'s indirect indifference curve at any point \( (m, g) \) in this plane after substituting the RPF (eq. (14)) into the indirect utility function is as follows:

\[
\left. \frac{\partial g}{\partial m} \right|_{V \text{ const.}} = -\left( \frac{V_m}{V_g} \right), \text{ where}
\]

\[
V_m = g^\beta \alpha^\alpha (1 - \alpha)^{(1 - \alpha)} \left[ (-1 + \gamma) y + \left( \frac{1 - 2(1 + \gamma)(1 + m)m - (1 + \gamma)m^2}{(1 - m)^2} \right) \tilde{p}(m)^{-\alpha} \right]
\]

\[
+ \left( y(1 - (1 + \gamma)m) + \left( \frac{m - (1 + \gamma)m^2}{(1 - m)} \right) \tilde{y} \right) (-\alpha) \tilde{p}(m)^{(-\alpha - 1)} \frac{\partial \tilde{p}(m)}{\partial m}
\]

\[
V_g = \beta g^{\beta - 1} \left[ \alpha^\alpha (1 - \alpha)^{(1 - \alpha)} \left( y(1 - (1 + \gamma)m) + \left( \frac{m - (1 + \gamma)m^2}{(1 - m)} \right) \tilde{y} \right) \tilde{p}(m)^{-\alpha} + \phi \right]
\]

\( \tilde{p}(m) = \) the equilibrium value of \( p \) given \( m \). That is \( \tilde{p}(m) \) is the median voter's preferred \( p \) for a given \( m \). Table 1 shows that \( \frac{\partial \tilde{p}(m)}{\partial m} < 0 \).

The following function indicates how the slope of an indifference curve changes with income:
\[
\frac{\partial^2 g}{\partial m \partial y} \bigg|_{V = \text{const.}} = \left\lbrack \frac{V_y m V_y - V_m V_y}{(V_y)^2} \right\rbrack
\]

where

\[
V_{my} = g^\beta \alpha^\alpha (1 - \alpha)^{(1 - \alpha)} \left[ -\left( 1 + \gamma \right) \tilde{p}(m)^{-\alpha} + (1 - (1 + \gamma)m) \left( -\alpha \right) \tilde{p}(m)^{(-\alpha - 1)} \frac{\partial \tilde{p}(m)}{\partial m} \right]
\]

(30)

\[
V_{gy} = \beta g^\beta \left( \alpha^\alpha (1 - \alpha)^{(1 - \alpha)} (1 - (1 + \gamma)m) \right) > 0.
\]

The sign of \( V_{my} \) depends on the value of the quantity in brackets [•], and thus depends only on \((m, \tilde{p}(m))\). The sign of \( V_{my} \) is stated in Lemma 1.

**Lemma 1:** *It can be shown computationally that the marginal utility of an increase in the income tax rate over the feasible values of \( m \) decreases as household income increases, i.e., \( V_{my} < 0 \) where \( m \in [0, 0.689] \).*

Let \( V_m(y) \) = the value of the derivative of \( V \) with respect to \( m \) when \((m, g) = \)(\( \tilde{m}, \tilde{g} \)) and income = \( y \). Let \( V_g(y) \) = the value of the derivative of \( V \) with respect to \( g \) when \((m, g) = \)(\( \tilde{m}, \tilde{g} \)) and income = \( y \). It can be shown that:

\[
V_m(\tilde{y}) > 0 \text{ and } V_g(\tilde{y}) > 0
\]

(31)

Based on (29), (30), Lemma 1, and (31), Figure 3 illustrates households’ indifference curves through the sequential equilibrium values of \( \tilde{m} \) and \( \tilde{g} \), or \( \tilde{y} \)’s most preferred \((m, g)\). Also illustrated in Figure 3 is the function \( \tilde{g}(m) \) which represents the relationship between the income tax rate determined in the first stage of voting and the equilibrium public good provision in the second stage of voting. Table 1 indicates that

\[
\frac{\partial \tilde{g}(m)}{\partial m} < 0 \text{ over all the feasible values of } m.
\]

The function \( \tilde{g}(m) \) is also drawn so that it is

36
Figure 3
tangent to median voter’s (\( \tilde{y} \)'s) indifference curve, which is labeled \( \tilde{V} \), at \( \tilde{y} \)'s most preferred \((m, g) = (\tilde{m}, \tilde{g})\).

Based on (29), (30), Lemma 1, and (31), the indifference curves of all households with \( y < \tilde{y} \) through the point \((\tilde{m}, \tilde{g})\) in Figure 3 are either positively sloped and concave, as represented by indifference curve \( V^1 \), or are negatively sloped and steeper than \( \tilde{V} \), as represented by \( V^2 \). As indicated in Figure 3, all these households with \( y < \tilde{y} \) prefer points on \( \tilde{g}(m) \) where \( m > \tilde{m}, g < \tilde{g} \), and consequently \( p < \tilde{p} \) since \( \frac{\partial \tilde{p}(m)}{\partial m} < 0 \).

Again based on (29), (30), Lemma 1, and (31), the indifference curves of all households with \( y > \tilde{y} \) through the point \((\tilde{m}, \tilde{g})\) in Figure 3 are either negatively sloped and flatter than \( \tilde{V} \), as represented by \( V^3 \), or positively sloped and convex, as represented by indifference curve \( V^4 \). As indicated in Figure 3, all these households with \( y > \tilde{y} \) prefer points on \( \tilde{g}(m) \) where \( m < \tilde{m}, g > \tilde{g} \), and consequently \( p > \tilde{p} \) since \( \frac{\partial \tilde{p}(m)}{\partial m} < 0 \).

3.5 Welfare and Distributional Effects of Redistribution Policy in the Single Community Case

As Table 1 shows, in the computed equilibrium when the income tax rate is equal to the \( MVE \) rate, \( \tilde{m} = 2\% \), the housing price and public good provision are lower and the property tax rate is slightly higher compared to the no redistribution case, \( m = 0\% \). Also, as measured by equivalent variation with \( m = 0 \) as the benchmark, 50.4% of households, or households with endowed income below $37,300, are better off when \( m = \tilde{m} = 2\% \). However, there is an aggregate welfare loss among households from \( m = 0 \) to \( \tilde{m} \) in that
the per capita equivalent variation is -$504.\(^\text{18}\) The per capita change in rents to absentee landlords is -$34, and hence the total per capita welfare loss is -$538.

Since utility is linear in income, the mean endowed income household’s (\(\bar{y}'s\)) most preferred redistribution policy maximizes per capita equivalent variation. Given the parameters of the computational model, \(\bar{y}\) most prefers \(m = 0\), and thus \(r = 0\). This is also the income tax rate and lump-sum grant that maximizes a utilitarian social welfare function with equal weights on the utility of all households. This result is in contrast to most of the results in the optimal taxation literature (Mirrlees 1971, Stern 1976). In these models labor distortions create a deadweight loss associated with redistribution policy, but the optimal level of redistribution as measured by a utilitarian social welfare function with equal weights is still positive. Optimal redistribution is positive in these models because the marginal utility of income is diminishing and redistributing from the rich to the poor is beneficial from the perspective of the utilitarian planner. In my model the marginal utility of income is constant, and thus the only potential benefit of redistributing from the rich to the poor from the perspective of the utilitarian planner is that it may cause the median voter to choose a higher level of public good provision closer the mean voter’s preferred level. However, the degree of the calibrated deadweight loss in the computations above causes \(g\) to fall as \(r\) increases, and thus redistribution reduces overall welfare.

\(^{18}\)I use equivalent variation simply because it is easier to calculate the effects of income changes on consumption. Clearly, very similar results would hold using compensating variation.
4. Multi-Community Equilibrium Conditions

In the multi-community case households are able to migrate from one community to another and locate in the jurisdiction they most prefer. The \( (g^j, p^j) \) combination chosen among the possible points on the BPF in each community is determined in the second stage of voting and the households are assumed to be “utility-taking” when voting. In the first stage of voting, the redistributive income tax rate and lump-sum grant \( (m,r) \) are determined by a vote of all residents of the jurisdiction of the higher level of government.

Equilibrium in the multi-community model expands on the single government equilibrium. The multi-community equilibrium is defined as an allocation of households to communities such that the following conditions hold:

1) **Internal Equilibrium in each community** \( j \) given the \( (m, r) \) pair:
   a. The housing market clears (eq. \((11)\)).
   b. The local government budget is balanced (eq. \((4)\)):
   c. There is a Majority-Rule Voting Equilibrium (MVE) in the second stage of voting over the public good expenditures and housing prices, \( (g^j, p^j) \), on each community’s BPF given \( (m,r) \).

2) **Intercommunity equilibrium** given the \( (m,r) \) pair:
   a. Each community is occupied
   b. No household want to move to another community

3) **Voting Equilibrium in first-stage of voting**
   a. The state government budget is balanced (eq. \((8)\)), which defines the RPF.
   b. There is Majority-Rule Voting Equilibrium (MVE) over the income tax rates and lump-sum grants \( (m,r) \) on the RPF in the first stage of voting.

The Internal Equilibrium conditions a. and b. above combined with the voters “utility-taking” assumption defines the BPF for a community. As mentioned above,
since preferences exhibit the single crossing property (SCP) in the \((g, p)\) space, \(\bar{y}'s\) most preferred point on the BPF in each community is the majority voting equilibrium in the second stage of voting.

Although equilibria in which two or more communities have the same level of provision of the public good cannot be ruled out, the focus of this paper is on Tiebout-type equilibria where \(g' \neq g'\) for all jurisdictions \(j \neq i\).\(^1\) For these types of equilibria, the SCP and the definition of intercommunity equilibrium above imply the following necessary properties of intercommunity equilibrium\(^2\).

Intercommunity Equilibrium Property 1: Stratification. Each community is formed of households with incomes in a single interval. If \(y\) and \(y'\) live in the same community, with \(y' > y\), then \(y'' \in [y, y']\) also lives in that community.

Intercommunity Equilibrium Property 2: Boundary Indifference. Communities can be ordered from lowest to highest income levels. When they are ordered this way, there is a "boundary" income between two successive communities. The "border" household (i.e., one with the boundary income) between any two adjacent communities is indifferent between the communities.

\(^1\) If \(g' = g'\), then \(p' = p'\), otherwise all households would prefer the community with lower \(p\) and the other community would not be occupied, contradicting intercommunity equilibrium condition a. I thus ignore as uninteresting equilibria, if there are any, in which two or more communities are replicas of each other with the same price of housing and public good provision, and thus all households are indifferent between these communities. These types of equilibria exist if voters are “myopic” and do not take into account household migration effects of changes in their respective community’s tax/expenditure policy. (Calabrese, et. al. (2011)) When households are “utility-takers” they take into account these household migration effects, and thus existence of equilibria with \(g' = g'\) and \(p' = p'\) do not necessarily exist and seem unlikely to exist.

\(^2\) For more details, see Epple et al. (1984)
Intercommunity Equilibrium Property 3: Ascending bundles. If \( y^j \) is the highest income in community \( j \) and \( y^i \) is the highest income in community \( i \), then, in equilibrium, \( p^j > p^i \) and \( g^j > g^i \) if \( y^j > y^i \).

Intercommunity Equilibrium Property 1, Stratification, implies that the Internal Equilibrium conditions a. and b. for community \( j \) can be written as follows:

a. The housing market clears:

\[
\omega^j h_d(p^j)(\bar{y}^j(1-(1+\gamma)m)+r) = h_s(p^j). \tag{32}
\]

where \( \omega^j \) is household population density in the community (households per unit of land area).

b. The local government budget is balanced:

\[
g^j = t^j p^j h_d(p^j)(\bar{y}^j(1-(1+\gamma)m)+r) \tag{33}
\]

Existence of equilibrium is not guaranteed, but is not unusual. I provide computed examples below. Multiplicity of equilibria can arise if housing supplies differ across jurisdictions. In the case of two jurisdictions having different housing supplies, then either might be the lower-\( g \) and poorer jurisdiction. With \( J \) jurisdictions each with different housing supply, then \( J! \) equilibria can arise, where any jurisdiction would be the poorest, any of the remaining the second poorest, and so on. Each of these \( J! \) would likely be characterized by different levels of state level redistribution.

4.1 Computationally Determining the Voting Equilibrium in the First-Stage of Voting.

In the single government case above, the MVE result in the first-stage of voting exploited the linearity of utility in income. In the multi-community case, the three intercommunity equilibrium properties above imply utility is not linear in income. Utility
is still linear in income within a community, but not across all the communities. A three-community group example of intercommunity equilibrium is represented graphically in Figure 4 for an arbitrary \((m,r)\) pair. The level of utility that households obtain in this example is represented by the utility function \(V(m,r)\). Based on Intercommunity Equilibrium Property 1: *Stratification* above, all \(y < y'\) occupy community 1, all \(y \in (y^1, y^2)\) occupy community 2, and all \(y > y^2\) occupy community 3. Consistent with Intercommunity Equilibrium Property 2: *Boundary Indifference*, endowed-incomes \(y^1\) and \(y^2\) are the boundary incomes. Based on Intercommunity Equilibrium Property 3: *Ascending bundles*, \(p^3 > p^2 > p^1\) and \(g^3 > g^2 > g^1\).

Since utility is not linear in income, it can not be shown analytically that the \(\tilde{y}\) in the state is pivotal in voting over the \((m,r)\) pairs on the RPF. The MVE over the RPF must be determined computationally by checking if one \((m,r)\) defeats all other \((m,r)\) pairs on RPF in pair-wise voting. In order to facilitate the computational check for the MVE on the RPF, note that the SCP of the preferences is due to the parameter \(\phi < 0\). As \(\phi \to 0\), all households preferences over \((g,p)\) pairs converge to the same ordering. These means if \(\phi = 0 - \varepsilon\), where \(\varepsilon\) is vanishingly infinitesimal, then in *intercommunity equilibrium* all households must be almost indifferent between all communities.\(^{21}\) If this is the case, utility is almost linear in income over all communities for any \((m,r)\) pair, and the \(\tilde{y}\) in the state would almost definitely be pivotal in voting. Hence, the first-step in computationally determining the MVE is to use \(\tilde{y}\)’s most preferred income tax/transfer

\(^{21}\) In fact, the utility-taking assumption implies that equilibrium does not exist where all households are indifferent to all communities. Hence, equilibrium does not exist if \(\phi = 0\).
FIGURE 4: A three-community group example of intercommunity equilibrium.
policy, \((\bar{m}, \bar{r})\), on the RPF as the first candidate for the MVE, and then check computationally that it defeats all other \((m, r)\) on the RPF in pair-wise voting.

In these multi-community computations, the parameter values for the utility function, housing supply function, and the distribution of income calibrated above in section 3.3 for the single-community case are used. I assume five local jurisdictions – a large city and four smaller suburbs that have equal area. The total land supply is normalized to 1. The city is assumed to have 40% of the total land area and each of the suburbs 15%. I assume that the city is the poorest jurisdiction. The jurisdictions are numbered from poorest to richest: Hence, \(L_1 = .4\), and \(L_2 = L_3 = L_4 = L_5 = .15\), where \(L_j\) equals community \(j\)’s land share. As shown above, the housing supply in community \(j\), \(C^j\), is:

\[
H_i^j(p_h^j) = L_i \left( p_h^j \right)^3
\]  

(34)

The median voter’s preferred \((\bar{m}, \bar{r}) = (4.05\%, \$2,174)\). A 4.05% state income tax rate is approximately mid-range of all state income tax rates. State income tax rates vary from 0% to the highest marginal rate of 9.5% in Vermont. Only seven states do not have an income tax, and two other states limit income tax to only dividends and interest income.\(^{22}\)

Table 2 shows the equilibrium values associated with the \((\bar{m}, \bar{r}) = (4.05\%, \$2,174)\) pair. The next step in computationally determining if \((\bar{m}, \bar{r})\) is the MVE is to vary \(m\) in very small increments over \([0, 0.689]\), where 0.689 is the maximum value of \(m\) given \(\gamma = 0.45\). The associated equilibrium for each of these alternative \(m\)’s are

Table 2: Multi-Community Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>((\bar{m}, \bar{r}) = (4.05%, $2,174))</th>
<th>((m, r) = (0%, $0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p^1)</td>
<td>$8.39</td>
<td>$8.01</td>
</tr>
<tr>
<td>(p^2)</td>
<td>$11.73</td>
<td>$11.52</td>
</tr>
<tr>
<td>(p^3)</td>
<td>$14.28</td>
<td>$14.18</td>
</tr>
<tr>
<td>(p^4)</td>
<td>$17.32</td>
<td>$17.34</td>
</tr>
<tr>
<td>(p^5)</td>
<td>$22.85</td>
<td>$23.08</td>
</tr>
<tr>
<td>(y^1)</td>
<td>$21,960</td>
<td>$21,290</td>
</tr>
<tr>
<td>(y^2)</td>
<td>$32,485</td>
<td>$31,642</td>
</tr>
<tr>
<td>(y^3)</td>
<td>$49,337</td>
<td>$48,251</td>
</tr>
<tr>
<td>(y^4)</td>
<td>$81,713</td>
<td>$80,267</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (y^1)</td>
<td>$22,843</td>
<td>$21,290</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (y^2)</td>
<td>$32,749</td>
<td>$31,642</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (y^3)</td>
<td>$48,610</td>
<td>$48,251</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (y^4)</td>
<td>$79,084</td>
<td>$80,267</td>
</tr>
<tr>
<td>(\tilde{y}^1)</td>
<td>$14,143</td>
<td>$13,814</td>
</tr>
<tr>
<td>(\tilde{y}^2)</td>
<td>$26,918</td>
<td>$26,181</td>
</tr>
<tr>
<td>(\tilde{y}^3)</td>
<td>$39,946</td>
<td>$39,012</td>
</tr>
<tr>
<td>(\tilde{y}^4)</td>
<td>$62,149</td>
<td>$60,941</td>
</tr>
<tr>
<td>(\tilde{y}^5)</td>
<td>$119,529</td>
<td>$117,826</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (\tilde{y}^1)</td>
<td>$15,485</td>
<td>$13,814</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (\tilde{y}^2)</td>
<td>$27,510</td>
<td>$26,181</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (\tilde{y}^3)</td>
<td>$39,771</td>
<td>$39,012</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (\tilde{y}^4)</td>
<td>$60,670</td>
<td>$60,941</td>
</tr>
<tr>
<td>After income tax and lump-sum transfer (\tilde{y}^5)</td>
<td>$114,678</td>
<td>$117,826</td>
</tr>
<tr>
<td>(N^1)</td>
<td>27.86%</td>
<td>26.70%</td>
</tr>
<tr>
<td>(N^2)</td>
<td>16.37%</td>
<td>16.36%</td>
</tr>
<tr>
<td>(N^3)</td>
<td>18.56%</td>
<td>18.78%</td>
</tr>
<tr>
<td>(N^4)</td>
<td>18.69%</td>
<td>19.10%</td>
</tr>
<tr>
<td>(N^5)</td>
<td>18.52%</td>
<td>19.06%</td>
</tr>
</tbody>
</table>

After income tax and lump-sum transfer

\(1\) \(\bar{m}\) = (4.05\%, $2,174) \(\bar{r}\) = (0\%, $0)
\[
\begin{array}{|c|c|c|}
\hline
& (\bar{m}, \bar{r}) = (4.05\%, 2174) & (m, r) = (0\%, 0) \\
\hline
\tilde{t}_1 = & 8.54\% & 7.72\% \\
\hline
\tilde{t}_2 = & 19.66\% & 18.86\% \\
\hline
\tilde{t}_3 = & 31.82\% & 30.83\% \\
\hline
\tilde{t}_4 = & 47.36\% & 46.20\% \\
\hline
\tilde{t}_5 = & 62.63\% & 61.50\% \\
\hline
\tilde{g}_1 = & $437 & $354 \\
\hline
\tilde{g}_2 = & $1,661 & $1,527 \\
\hline
\tilde{g}_3 = & $3,543 & $3,394 \\
\hline
\tilde{g}_4 = & $7,263 & $7,179 \\
\hline
\tilde{g}_5 = & $19,770 & $20,187 \\
\hline
\end{array}
\]

Note: \( \bar{p}_j \) is the gross of property tax price of housing in community \( j (j = 1,2,3,4,5) \), \( \bar{y}_j \) is the highest endowed income in community \( j \), \( \bar{\bar{y}}_j \) is the median endowed income in community \( j \), \( \bar{\bar{\bar{y}}}_j \) is the mean income in community \( j \), \( \bar{N}_j \) is community \( j \)’s share of the total population, \( \tilde{t}_j \) is community \( j \)’s property tax rate, and \( \tilde{g}_j \) is community \( j \)’s per capita expenditure on the public good.
computed. Then the vote favoring \( \tilde{y} \)'s ideal point, \( \tilde{m} \), relative to each of these alternative points is calculated. Figure 5 presents the results of these comparisons. The vertical axis represents the percentage of households favoring \( \tilde{m} \) to the corresponding alternative \( m \) on the horizontal axis. As is evident from the graph, the median’s ideal point, \( (\tilde{m}, \tilde{r}) \), defeats all other \( (m, r) \) pairs on the RPF.

4.2 Effects of Redistribution Policy in the Multi-Community Case

A comparison of the simulation results for equilibrium (\( \tilde{m} = 4.05\% \)) and simulated results for \( m = 0 \) are presented in Table 2. As the results in the table indicate, introducing income tax financed redistribution at the state level causes a migration “downward” across communities. For example, the relatively poorer households in the richest community, \( C^5 \), migrate to \( C^4 \). This is because households with \( y > $36,980 \) have lower disposable income after the income tax/transfer policy is introduced. Hence, all households originally in \( C^5 \) have lower disposable income, causing the relatively poorer households in \( C^5 \) to migrate to \( C^4 \), which has a lower housing price. The same effect causes the poorer households in \( C^4 \) to migrate to \( C^3 \). Both \( C^5 \) and \( C^4 \) lose population and aggregate disposable income, resulting in a reduction in the gross of tax housing prices in both communities even though the property tax rate increases in both communities. The degree of the loss of aggregate income in \( C^5 \) offsets the higher property tax rate, and \( g^5 \) decreases. The loss of aggregate income in \( C^4 \) is not as great, and \( g^4 \) increases with the increase in \( t^4 \).

Even though the majority of residents of \( C^3 \) lose from the lump-sum redistribution
Figure 5: Vote for Median Ideal $m = 4.05\%$ Against All Other Feasible $m$
and the total population decreases, the in-migration of relatively richer households from $C^4$ and the out-migration of relatively poorer households into $C^2$ generates higher after-transfer aggregate income. All the residents of $C^2$ and $C^1$ gain from the lump-sum transfer, and combined with an increase in population and relatively richer households moving into these two communities (and relatively poorer ones moving out of $C^2$), after-transfer aggregate income also increases in these two lowest income communities. The greater aggregate income and median voter income in each of these three lowest income communities induces higher housing prices, public good provision, and tax rates in each community. These effects are in contrast to the effects of redistribution policy in the single-community model, where increases in redistribution lead to lower $(g, p)$. Also, since the level of public good provision increases in all four lowest-income communities, over 80% of the population experiences higher levels of public good provision when the state provides a lump-sum redistribution, even with a significant deadweight loss associated with the income tax used to finance the lump-sum transfer.

Using equivalent variation to measure the change in welfare from no redistribution ($m = 0$) to the equilibrium level of redistribution $(\bar{m}, \bar{r}) = (4.05\%, $2,174), all $y < $37,900, or 51% of all households, are better-off. Just as in the single community case, however, there is an aggregate welfare loss among households. The per capita equivalent variation is -$956 and the per capita change in rents to absentee landlords is -$63.

Since utility is not linear in income, unlike in the single community case, the mean endowed income household’s $(\bar{y}'s)$ most preferred redistribution policy does not necessarily maximizes per capita equivalent variation. However, again $m = 0$ maximizes
5. Contrasting Effects of Redistribution Policy in the Single-Community and Multi-Community Cases

A striking contrast between the computed single-community equilibrium and the multi-community equilibrium is the significantly higher lump-sum transfer and income tax rate in the multi-community equilibrium. In the single-community case \((\bar{m}, \bar{r}) = (2\%, \$1,084)\), and in the multi-community case \((\bar{m}, \bar{r}) = (4.05\%, \$2,174)\). Since the pivotal household is the same in either case, a federal system with an income stratified population creates incentives for \(\bar{y}\) to prefer a higher level of income tax financed redistribution. One possible explanation for this result is that \(\bar{y} = \$36,942\) locates in \(C^3\) in the multi-community equilibrium. As shown in Table 2 and discussed in the previous section, state redistribution increases aggregate income in \(C^3\). In the single-community case, redistribution reduces aggregate income because of the deadweight loss associated with income tax. Since aggregate income increases in \(C^3\) and the income of the intracommunity pivotal voter on \(C^3\)’s BPF increases, redistribution increases \(g^3\).

Redistribution always reduces public good provision in the single-community case. In the multi-community equilibrium, \(\bar{y}\)’s after tax/transfer income (\$36,944) is only slightly greater than when there is no redistribution (\$36,942). However, \(\bar{y}\) has access to more public good provision as \(g^3\) increases from \$3,394 to \$3,543 with only a small concomitant increase in \(p^3\).
In the public finance literature it is generally the result that the rich gain from decentralization while the poor lose. (Calabrese, Cassidy, and Epple (2002), Calabrese, Epple, and Romano (2007), Oates (1972)). It is thought that through decentralization the rich can segregate themselves from the poor, reducing the ability of the poor to free-ride on the rich in financing public expenditures. In the model in this paper when a “central” government redistributes income, a federal system resulting in the population stratified by income leads to more redistribution than in a unitary state. Hence, the distributional consequences are reversed. All households with endowed income below $23,510, or 31% of the population, are better-off in a federation than in a unitary state. The distributional effects are opposite for \( y > 23,510 \).\(^{23}\)

6. Conclusion

In this paper, I have characterized equilibrium conditions for a model with income taxed financed redistribution and property tax financed public good provision. I characterized equilibrium for a unitary state and for an economic federation with two levels of government. In both types of government structures, it is assumed there is a two-stage voting process in which the redistribution level financed by income tax is determined first and then the public good levels financed by property taxes are voted on. In the federation model, there is one central government that redistributes income and multiple local governments, each of which provides a public good. Unlike previous hierarchical government models, voters understand the interrelationship between the fiscal policies of the two levels of government. Voters are able to anticipate how

\(^{23}\) Calabrese, Epple, and Romano (2012) find that in general with the “Utility-Taking” assumption and without redistribution, virtually all households are made worse off in the Multi-community equilibrium compared to the Single-Community Equilibrium.
changing the central government’s fiscal policy effect housing markets and the amount of public good provision in the local communities. Also unlike previous models, when voting at the local levels, households have some sophistication in predicting how changing their respective local community’s property tax rate and public good provision impacts the migration of households into and out of their community, and the demand for housing of current residents of their community.

In the unitary state, or single community model, I show analytically that non-distortionary income tax financed redistribution causes the pivotal voter to prefer a higher level of property tax financed public good provision. In a computed model with parameters, including a deadweight loss parameter, calibrated to realistic values, introducing redistribution leads to a majority voting equilibrium with lower public good provision and housing prices. Aggregate welfare falls if there is centralized redistribution, even though a majority of households are made better off.

In contrast to the unitary state model, centralized redistribution results in higher levels of public good provision in almost all the local communities. However, the welfare and distributional affects of redistribution are similar to the unitary state model.

The model in this article is also used to analyze the effects of decentralizing public good provision when there is centralized redistribution. Computational equilibria indicate that the income tax rate and the level of redistribution is greater with a federal system than a unitary state. The higher level of redistribution under a federal system is a critical reason that the usual distributional impacts of decentralization are reversed; that is, lower income household benefit and the higher income households are made worse-off by decentralization of public good provision.
I believe the results in this paper open the door to important variations and extensions. One possible variation is to replace the state income tax with a tax on numeraire consumption. Also, instead of the state redistributing income it could provide a public good. It is also possible to add local income taxes and redistribution to the model. Potentially, the two-stage voting model in this paper could be applied to develop a model with state income and sales taxes financing redistribution and a public good, and local property, income, and head taxes financing redistribution and public goods.

Other possible extensions include investigating how varying the sophistication of voters at both the state and local levels impact equilibrium values and the welfare and distributional consequences. It is possible to model state voters who are myopic about the impact of state redistribution on local housing prices and public good levels. Also, when voting the tax/expenditure in their local communities, voter sophistication can vary from extremely myopic, in which they assume housing demands are fixed in all communities, to highly sophisticated voters who are able to predict the general equilibrium impacts of changes in his/her community’s tax/expenditure policy on all communities.
References


