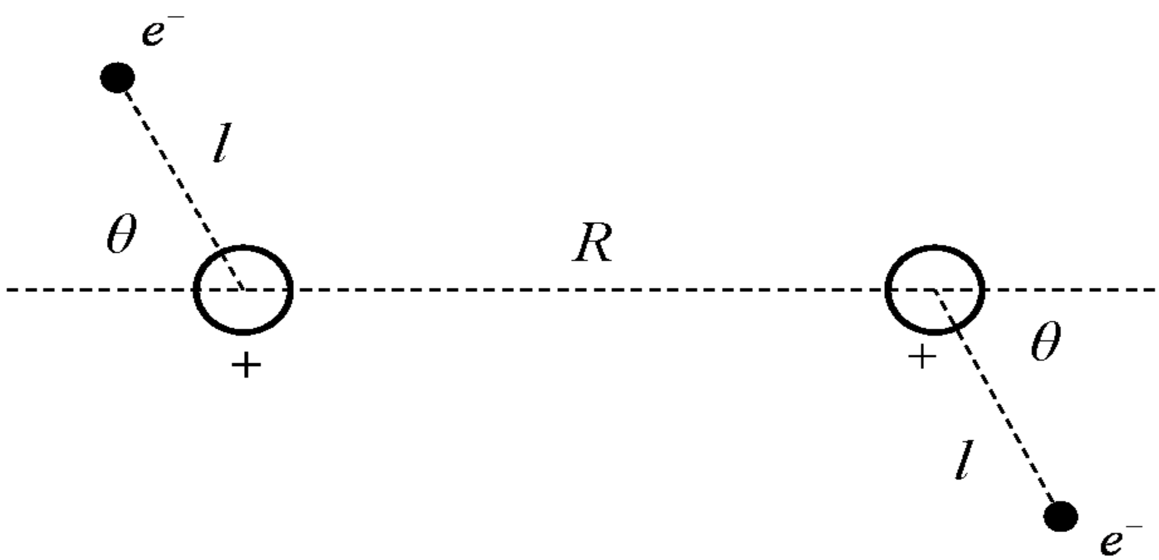
V25.0109: General Chemistry I: Honors

Problem set #3: due 10/2

Practice problems from Chapter 2: 15,25,27,29,30

Graded problems



1. The total Coulomb potential energy is an example of what is known as a *pairwise additive potential energy.* This means that the total Coulomb potential energy of a system of N charged particles is the sum of the Coulomb potential energies between all possible unique pairs of charges that exist among N charges.

   The figure above shows a snapshot of an H_2 molecule, which consists of two protons (indicated by the "+" sign) and two electrons (indicated as e^-). The two protons are a distance R apart, and each electron is a distance l from one of the protons. The line joining each electron to its closest proton makes an angle θ with the line joining the two protons (see figure for clarification).

   a. Assuming that all particles in the molecule can be treated as simple point charges, write down the total Coulomb energy of this system in terms of R, θ, l, and the fundamental constants e and ϵ_0.

   b. What is the actual value of the total Coulomb energy in Joules/mol if $R = 0.9$ Å, $l = 0.5$ Å, and $\theta = 40°$.

2. a. Photons of frequency $1.12{\times}10^{15}$Hz impinge on a metal surface of unknown work function. It is observed that electrons are ejected with residual kinetic energy. The electron stream is sent through a region with perpendicular electric and magnetic fields. The electrons enter this region with their velocity perpendicular to both the electric and magnetic fields. It is found that when the electric field is tuned to a value of $3.06{\times}10^6$ V/m and the magnetic field is tuned to a value of 3.51 Tesla (the SI unit of the magnetic field), the stream of electrons passes through the field region completely undeflected. What is the work function of the metal?

b. A photon of frequency 1.8×10^{16}Hz strikes the electron of a hydrogen atom in its ground energy level as prescribed by the Bohr formula. If a photosensitive detector is used to detect whether or not the atom is ionized, will detector register an electron? If so, with what velocity (in m/s) will the electron strike the detector?

3. Given a continuous random variable $x \in [a, b]$, the probability that x has a value in a small interval dx is given by the formula

$$\text{Probability that } x \text{ is in } dx = P(x)dx$$

where $P(x)$ is called the *probability density* or *probability distribution* of x.

a. The function $P(x)$ must satisfy two conditions:

$$P(x) \geq 0 \qquad \text{for all } x$$

$$\int_a^b P(x)dx = 1$$

Explain why these two conditions must be satisfied.

b. The second condition in part a is called the *normalization condition* on $P(x)$. Suppose you are given $P(x)$ in the form

$$P(x) = e^{-ax}$$

for $0 \leq x < \infty$. Does this $P(x)$ satisfy the normalization condition? If not, how would you need to modify $P(x)$ so that it does satisfy this condition?

c. In class, we discussed the concept of a probability amplitude $A(x)$, where $A(x)$ is complex and is related to the probability distribution by

$$P(x) = A^*(x)A(x) = |A(x)|^2$$

For the probability distribution in part b, is there a unique formula for $A(x)$? If so, determine the unique expression for $A(x)$. If $A(x)$ is not unique, give at least four possible expressions for $A(x)$.