V25.0109: General Chemistry I: Honors

Problem set #2: due 9/25

Practice problems from Chapter 2: 1,3,6,7,8

Graded problems

1. The aspirin substitute, acetaminophen ($C_8H_9O_2N$), is produced by the following three-step synthesis:

   I. $C_6H_5O_3N(s) + 3H_2(g) + HCl(aq) \longrightarrow C_6H_8ONCl(s) + 2H_2O(l)$

   II. $C_6H_8ONCl(s) + NaOH(aq) \longrightarrow C_6H_7ON(s) + H_2O(l) + NaCl(aq)$

   III. $C_6H_7ON(s) + C_4H_6O_3(l) \longrightarrow C_8H_9O_2N(s) + HC_2H_3O_2(l)$

   The percentage yields by mass of steps I,II, and III are 87%, 98% and 92%, respectively. Suppose 800 g of $C_6H_5O_3N$, 40g of H_2 gas and 300 g or HCl are combined for step I. After completion of step I, 300 g of NaOH are added for step II, and after completion of this step, 510 g $C_4H_6O_3$ are added for step III. How many grams acetaminophen will be produced in the final step? What is the overall percentage yield by mass of the reaction?

2. Bromine and Fluorine gases react to produce BrF_5 according to the equation:

   $$Br_2(g) + 5F_2(g) \longrightarrow 2BrF_5(g)$$

   Initially, the reaction vessel contains 1.66 moles of the reactant gases, but the individual amounts of bromine and flourine gases are unknown. The vessel is heated, and the gases react as prescribed in the above equation. When all reactions have ceased, there are 1.35 moles of gas in the vessel. The final contents of the reaction vessel are analyzed and found to contain *both* BrF_5 *and* bromine gas. Determine the *initial* numbers of moles of bromine and fluorine gases originally present in the container before heating.

3. Xenon tetrafluoride can be hydrolyzed to give xenon trioxide according to the (unbalanced) reaction:

   $$XeF_4(s) + H_2O(l) \longrightarrow XeO_3(aq) + Xe(g) + O_2(g) + HF(aq)$$

   Here the designators (s), (l) (aq), and (g) indicate solid, liquid, dissolved and gaseous species, respectively.

   a. In how many ways can this reaction be balanced? Write down a balanced reaction in which the stoichiometric coefficients sum to 51.

   b. Suppose an unknown amount of XeF(s) is dissolved in a large beaker of water. After the solid completely reacts, it is found that 0.08 total moles of gas are produced in the reaction. Determine the original mass of the solid xenon tetrafluoride.

4. Consider the general reaction

   $$a\text{A} + b\text{B} \longrightarrow c\text{C} + d\text{D}$$

   where A, B, C, and D are known compounds with molar masses M_{A}, M_{B}, M_{C} and M_{D}, respectively. Suppose amounts m_{A} and m_{B} of A and B (in grams) are combined and allowed to react. Assume that the reaction above is balanced as written and that the following inequality

   $$\frac{m_{\text{B}}}{bM_{\text{B}}} < \frac{m_{\text{A}}}{aM_{\text{A}}}$$

   holds. If no C and D are initially present, determine how much (in grams) C and D will be produced and how much (in grams) A and B will remain after the reaction has ceased.

5. a. Masses of atoms and molecules are often determined by the technique of mass spectrometry, which is based on the nature of the motion of charged particles in a magnetic field.

A charged particle moving with velocity $\mathbf{v}$ in a constant, uniform magnetic field $\mathbf{H}$ will experience a force $\mathbf{F}$ due to the field that is perpendicular to the directions of both $\mathbf{v}$ and $\mathbf{H}$ (see Fig. 1).

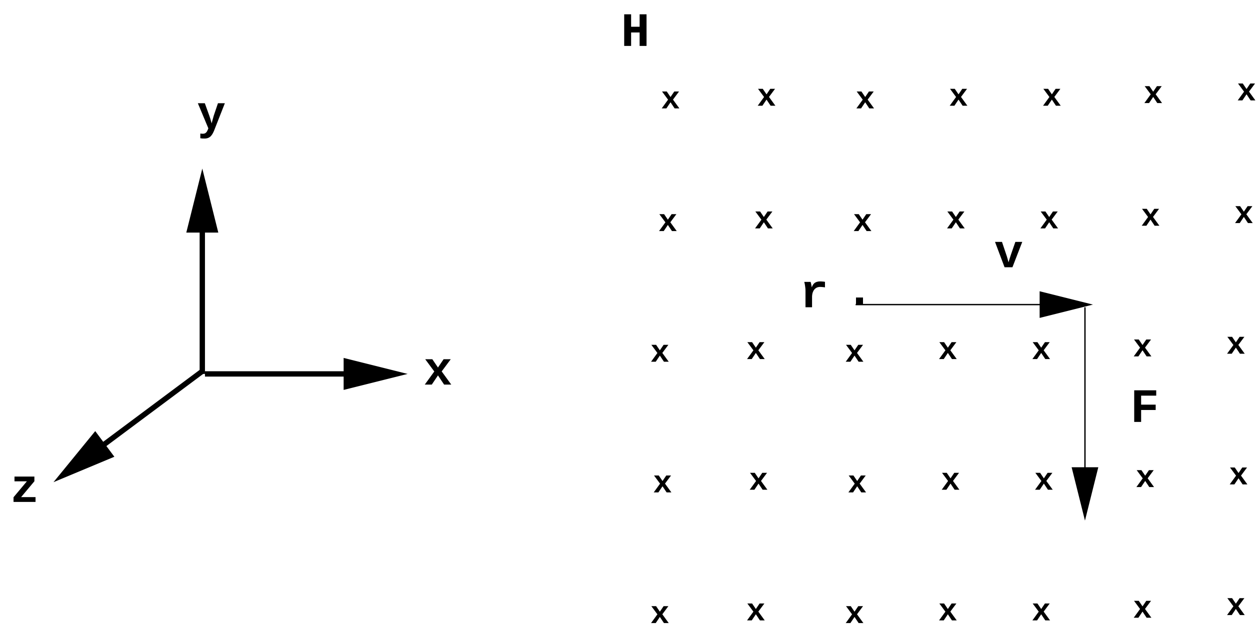

FIG. 1.

This force causes $\mathbf{v}$ to change in time, so that both $\mathbf{v}$ and $\mathbf{F}$ are changing. In the figure, the magnetic field is directed into the plane of the page. The particle is *initially* at a position $\mathbf{r}$, moving *initially* to the right with velocity $\mathbf{v}$ so that the force is *initially* down.

Simple geometrical vector addition can be used to deduce the motion of the particle. Recall that two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ can be added geometrically by placing the tail of one vector at the head of the other. The vector that completes the triangle is the sum $\mathbf{v}_1 + \mathbf{v}_2$ as shown in Fig. 2

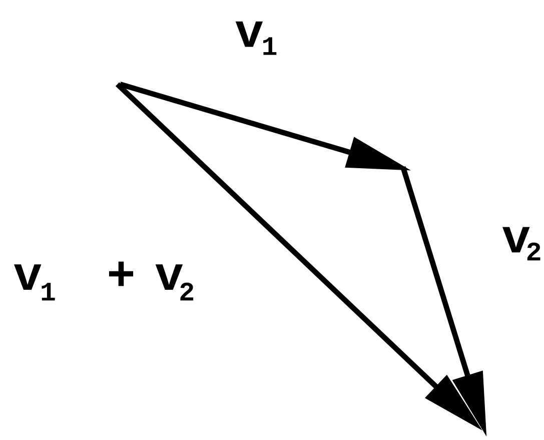

FIG. 2.

Given an initial velocity $\mathbf{v}(0)$ and force $\mathbf{F}(0)$, the velocity $\mathbf{v}(\Delta t)$ a short time Δt later can be calculated approximately by assuming the force is constant over the time interval. Note that $\Delta t > 0$ and $\Delta t \ll 1$. This means that the velocity at time Δt will be given in terms of $\mathbf{v}(0)$ and $\mathbf{F}(0)$ by the formula:

$$\mathbf{v}(\Delta t) = \mathbf{v}(0) + \frac{\Delta t}{m}\mathbf{F}(0)$$

and the new position $\mathbf{r}(\Delta t)$ is given approximately by

$$\mathbf{r}(\Delta t) = \mathbf{r}(0) + \Delta t \mathbf{v}(0)$$

Given the new velocity $\mathbf{v}(\Delta t)$, the new force $\mathbf{F}(\Delta t)$ can be determined, and these quantities fed in as new "initial conditions" to the above procedure. By iterating in this fashion, positions and velocities can be computed approximately at times Δt, $2\Delta t$, $3\Delta t$,...

Using simple geometrical vector addition and the above iterative procedure, deduce the nature of the of the motion of the charged particle in the magnetic field $\mathbf{H}$. Show *all* of your work and give a brief description of the particle's trajectory.

b. A particle of charge q moving in a region of uniform magnetic field $\mathbf{H}$ experiences a force whose magnitude is qvH. This force will give rise to a centripetal acceleration so that, by Newton's second law:

$$qvH = \frac{mv^2}{R}$$

where R is the radius of the particle's orbit, m is its mass, and v is the magnitude of its velocity upon entering the region of the magnetic field.

Consider a charged particle, initial at rest, that is accelerated through an electric field $\mathbf{E} = (E, 0, 0)$ over a distance l. The particle then enters a region of uniform magnetic field $\mathbf{H}$ perpendicular to its velocity. Show that its mass m is given by

$$m = \frac{qH^2R^2}{2El}$$

where R is the radius of its orbit.

6. Suppose, in second part of the J. J. Thompson experiment, we switch off the electric field instead of the magnetic field. That is, we propose to use the motion of a charged particle in the magnetic field to determine the charge-to-mass ratio e/m. A sketch of the field region is shown in the figure below:

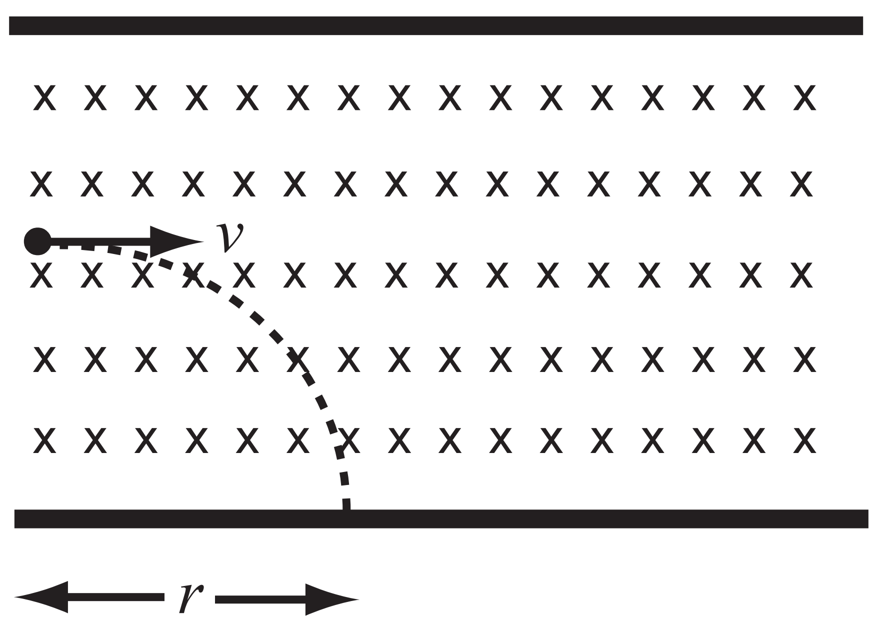

FIG. 3.

The point at which the electron enters the field is taken to be the origin of the coordinate system, and the electron's velocity is initially purely in the x-direction, with $v = E/H$, as determined from the first part of the experiment. Under the influence of the magnetic force, the elecron follows a circular path described by the equations

$$x(t) = \frac{v}{\omega} \sin \omega t$$

$$y(t) = \frac{v}{\omega} (\cos \omega t - 1)$$

where

$$\omega = \frac{eH}{m}$$

The electron strikes the lower plate at a distance r from its entry point, as shown in the figure. Derive an expression for the charge-to-mass ratio e/m in terms of E, H, and r when the experiment is carried out in this way.

7. a. Prove the following identity for geometric sums:

$$\sum_{n=0}^{\infty} n r^n = \frac{r}{(1-r)^2}$$

Hint: Try starting with the basic identity

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

and taking a derivative of the form $r\frac{d}{dr}$ of both sides.

b. In Planck's resolution of the ultraviolet catastrophe paradox, Planck proposed that the energies of an oscillator could take on the discrete values $E_n = nh\nu$. In fact, these energies are actually given by the formula

$$E_n = \left(n + \frac{1}{2}\right) h\nu$$

Using the result of part a, derive an expression for the average energy $\bar{E}$ of a large collection of oscillating charged particles. The average energy is given by

$$\bar{E} = \frac{\sum_{n=0}^{\infty} E_n e^{-E_n/k_{\rm B}T}}{\sum_{n=0}^{\infty} e^{\frac{-E_n}{k_{\rm B}T}}}$$