

Latent Variable Interaction and Quadratic Effect Estimation: A Two-Step Technique Using Structural Equation Analysis

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The author proposes an alternative estimation technique for latent variable interactions and quadratics. Available techniques for specifying these variables in structural equation models require adding variables or constraint equations that can produce specification tedium and errors or estimation difficulties. The proposed technique avoids these difficulties and may be useful for EQS, LISREL 7, and LISREL 8 users. First, measurement parameters for indicator loadings and errors of linear latent variables are estimated in a measurement model that excludes the interaction and quadratic variables. Next, these estimates are used to calculate values for the indicator loadings and error variances of the interaction and quadratic latent variables. Then, these calculated values are specified as constants in the structural model containing the interaction and quadratic variables.

Interaction and quadratic effects are routinely reported for categorical independent variables (i.e., in analysis of variance) frequently to aid in the interpretation of significant main effects. However, interaction and quadratic effects are less frequently reported for continuous independent variables.

Researchers have called for the inclusion of interaction and quadratic variables in models with continuous independent variables (Aiken & West, 1991; Cohen & Cohen, 1975, 1983; Jaccard, Turrisi, & Wan, 1990). However until recently there has been no adequate method of estimating interaction and quadratic effects among latent variables in structural equation models. Kenny and Judd (1984) proposed that interaction and quadratic latent variables could be adequately specified using products of indicators, under certain conditions. They demonstrated their proposed technique using COSAN (McDonald, 1978; now available in the SAS procedure CALIS) because it accommodates nonlinear constraints.

Hayduk (1987) subsequently implemented the Kenny and Judd (1984) technique using LISREL. However, the Hayduk approach required the specification of many additional latent variables to account for the loadings and error variances of the nonlinear indicators. The Kenny and Judd technique also required the creation of many additional variables, although fewer than the Hayduk approach.

Recently, LISREL 8 provided a nonlinear capability that can be used to implement the Kenny and Judd (1984) technique. However, this nonlinear capability may require the specification of many constraint equations, and it may create many COSAN-like variables using partial derivatives of the constraint equations.

As a result, the available techniques may produce specifica-

tion tedium, errors and estimation difficulties in larger structural equation models.

Objective and Overview

This article proposes an alternative to the Hayduk (1987) and the Kenny and Judd (1984) techniques. Because it creates no additional variables or equations, the proposed technique may be useful to LISREL 8 users with larger structural equation models or models with many indicators. Because it uses constants for the indicator loadings and error variances of interaction and quadratic latent variables, it may be appropriate for EQS and LISREL 7, which do not directly model these latent variables.¹

The proposed technique is implemented in two steps. For indicators in mean deviation form, loadings and error variances for the indicators of linear latent variables are estimated in a first-step measurement model.² Then, the nonlinear indicators of interaction and quadratic latent variables are created as products of the indicators of linear latent variables, as Kenny and Judd (1984) suggested. Next, the loadings and error variances for these product indicators are calculated using the first-step measurement model estimates, plus equations derived from Kenny and Judd's results. Finally, the relations among the linear, interaction, and quadratic latent variables are estimated, using a second-step structural model in which these calculated loadings and error variances are specified as constants. The balance of the article describes this technique.

¹ LISREL 8 is currently available for IBM-compatible personal computers only. When this article was written, Scientific Software International and SPSS, Inc. had no release date for a mainframe version of LISREL 8. As a result, LISREL 7 is still in use.

² An indicator in mean deviation form is the result of subtracting the mean of the indicator from the value of that indicator in each case. The resulting indicator has a mean of zero (see Aiken & West, 1991; Bollen, 1989, p. 13; Jaccard, Turrisi, & Wan, 1990, p. 28; Kenny & Judd, 1984).

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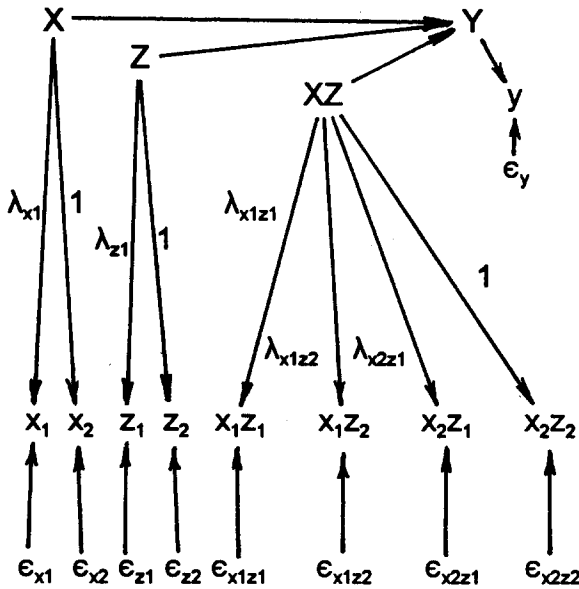


Figure 1. A covariance structure model.

Quadratic and Interaction Effect Estimation

For latent variables X and Z with indicators $x_1, x_2, z_1,$ and z_2 , Kenny and Judd (1984) proposed the interaction latent variable XZ could be specified with the product indicators $x_1z_1, x_1z_2, x_2z_1,$ and x_2z_2 . They also showed that the variance of product indicators such as x_1z_1 depends on measurement parameters associated with X and Z. Assuming that each of the latent variables X and Z is normally distributed and independent of the errors ($\epsilon_{x1}, \epsilon_{x2}, \epsilon_{z1},$ and ϵ_{z2} ; X and Z may be correlated), that the errors are mutually independent, and that the indicators and the errors are normally distributed and in mean deviation form (i.e., have a mean of zero), the variance of the product indicator x_1z_1 is given by

$$\begin{aligned} \text{Var}(x_1z_1) &= \text{Var}[(\lambda_{x1}X + \epsilon_{x1})(\lambda_{z1}Z + \epsilon_{z1})] \\ &= \lambda_{x1}^2 \lambda_{z1}^2 \text{Var}(XZ) + \lambda_{x1}^2 \text{Var}(X) \text{Var}(\epsilon_{z1}) \\ &\quad + \lambda_{z1}^2 \text{Var}(Z) \text{Var}(\epsilon_{x1}) + \text{Var}(\epsilon_{x1}) \text{Var}(\epsilon_{z1}) \quad (1) \\ &= \lambda_{x1}^2 \lambda_{z1}^2 [\text{Var}(X) \text{Var}(Z) + \text{Cov}^2(X,Z)] \\ &\quad + \lambda_{x1}^2 \text{Var}(X) \text{Var}(\epsilon_{z1}) + \lambda_{z1}^2 \text{Var}(Z) \text{Var}(\epsilon_{x1}) \\ &\quad + \text{Var}(\epsilon_{x1}) \text{Var}(\epsilon_{z1}), \quad (2) \end{aligned}$$

for x_1 and z_1 with expected values of zero. In Equations 1 and 2, λ_{x1} and λ_{z1} are the loadings of x_1 and z_1 on X and Z; ϵ_{x1} and ϵ_{z1} are the error terms for x_1 and z_1 ; $\text{Var}(X), \text{Var}(Z), \text{Var}(x_1z_1), \text{Var}(\epsilon_{x1}),$ and $\text{Var}(\epsilon_{z1})$ are the variances of X, Z, $x_1z_1, \epsilon_{x1},$ and ϵ_{z1} , respectively; and $\text{Cov}(X,Z)$ is the covariance of X and Z.

In the quadratic case (where $X = Z$), the variance of the product indicator x_1x_1 is given by

$$\begin{aligned} \text{Var}(x_1x_1) &= \text{Var}[(\lambda_{x1}X + \epsilon_{x1})(\lambda_{x1}X + \epsilon_{x1})] \\ &= \lambda_{x1}^2 \lambda_{x1}^2 \text{Var}(X^2) + 4\lambda_{x1}^2 \text{Var}(X) \text{Var}(\epsilon_{x1}) \\ &\quad + \text{Var}(\epsilon_{x1}) \quad (3) \\ &= 2\lambda_{x1}^2 \lambda_{x1}^2 \text{Var}^2(X) + 4\lambda_{x1}^2 \text{Var}(X) \text{Var}(\epsilon_{x1}) \\ &\quad + 2\text{Var}^2(\epsilon_{x1}). \quad (4) \end{aligned}$$

Table 1
Artificial Data Set Population Characteristics

Parameter	Variance	Value
Quadratic term model population		
X	1.00	
ϵ_{x1}	0.15	
ϵ_{x2}	0.55	
E_Y	0.20	
λ_{x1}		1.00
λ_{x2}		0.60
$\gamma_{Y,X}$		0.25
$\gamma_{Y,XX}$		-0.50
Interaction term model population		
X	2.15	
Z	1.60	
$\psi_{X,Z}$	0.20	
ϵ_{x1}	0.36	
ϵ_{x2}	0.81	
ϵ_{z1}	0.49	
ϵ_{z2}	0.64	
E_Y	0.16	
λ_{x1}		1.00
λ_{x2}		0.60
λ_{z1}		1.00
λ_{z2}		0.70
$\gamma_{Y,X}$		-0.15
$\gamma_{Y,XZ}$		0.70
$\gamma_{Y,Z}$		0.35

Note. For the quadratic term model population parameter: $Y = \gamma_{Y,X}X + \gamma_{Y,XX}X^2 + E_Y; x_1 = \lambda_{x1}X + \epsilon_{x1}; x_2 = \lambda_{x2}X + \epsilon_{x2}$. For the interaction term model population parameter: $Y = \gamma_{Y,X}X + \gamma_{Y,Z}Z + \gamma_{Y,XZ}XZ + E_Y; x_1 = \lambda_{x1}X + \epsilon_{x1} + x_2 = \lambda_{x2}X + \epsilon_{x2}; z_1 = \lambda_{z1}Z + \epsilon_{z1}; z_2 = \lambda_{z2}Z + \epsilon_{z2}$.

Kenny and Judd (1984) then specified Equations 2 and 4 using COSAN by creating additional variables for the terms in these equations. For example, Equation 2 required five additional variables, one each for $\lambda_x \lambda_z, \text{Var}(X) \text{Var}(Z) + \text{Cov}^2(X,Z), \text{Var}(X) \text{Var}(\epsilon_z), \text{Var}(Z) \text{Var}(\epsilon_x),$ and $\text{Var}(\epsilon_x) \text{Var}(\epsilon_z)$. These additional variables were then specified (constrained) to equal their respective Equation 2 terms for COSAN estimation. Some creativity is required, however, to estimate Equations 1 and 3 with EQS and LISREL 7 because these software products are not able to specify the nonlinear (product) terms in Equation 1 or 3.³

Hayduk's (1987) contribution was to provide a LISREL implementation of the Kenny and Judd (1984) technique. Hayduk's approach was to create additional latent variables to specify, for example, the right-hand side of Equation 2. It is difficult to do justice to Hayduk's approach in a few sentences, so the interested reader is directed to Hayduk's chapter 7 for details. In summary, to specify the first term of Equation 2, Hayduk created a chain of additional latent variables that affected the indicator x_1z_1 . Using three additional chains of latent variables, the remaining three terms in Equation 2 can be specified.

³ LISREL 8 provides constraint equations that can be used to implement the Kenny and Judd (1984) technique. Equation 1, for example, can be specified using two constraint equations: one for $\lambda_{x1} \lambda_{z1}$ and one for the balance of Equation 1 after the $\lambda_{x1}^2 \lambda_{z1}^2 \text{Var}(XZ)$ term.

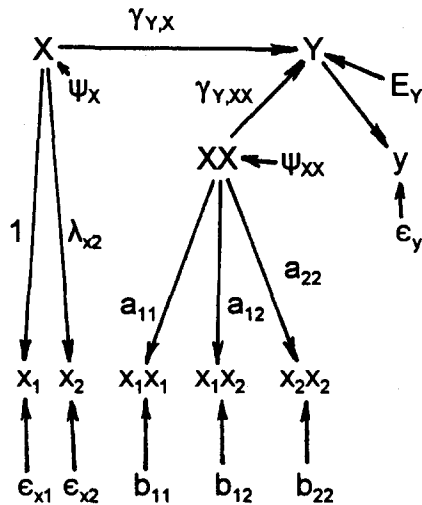


Figure 2. A quadratic model using the proposed approach. $a_{11} = \lambda_{x1}\lambda_{x1} = 1$, $a_{12} = \lambda_{x1}\lambda_{x2} = \lambda_{x2}$, $a_{22} = \lambda_{x2}\lambda_{x2}$; $b_{11} = 2\lambda_{x1}X\epsilon_{x1} + \epsilon_{x1}^2$; $b_{12} = \lambda_{x1}X\epsilon_{x2} + \lambda_{x2}X\epsilon_{x1} + \epsilon_{x1}\epsilon_{x2}$; $b_{22} = 2\lambda_{x2}X\epsilon_{x2} + \epsilon_{x2}^2$.

For a latent variable with many indicators, or for a model with several interaction or quadratic latent variables, the Hayduk (1987) approach of adding variables is arduous. For example, the single interaction model shown in Figure 1 requires an additional 30 latent variables to specify the loadings and error variances of the indicators for XZ.

As a result, researchers may find the specification of the additional variables or constraint equations required by the Hayduk (1987) and Kenny and Judd (1984) techniques difficult.⁴ The number of additional variables required or generated by these techniques may also lead to estimation difficulties produced by the large matrices required to specify these additional variables.

The next section proposes a technique that requires the specification of no additional variables or constraint equations.

A Proposed Estimation Technique

In estimating structural equation models, J. C. Anderson and Gerbing (1988) proposed that the measurement specification of the model should be assessed separately from its structural specification to ensure the unidimensionality of each of the latent variables in the model. This, the authors argued, avoids *interpretational confounding* (Burt, 1976), the interaction of the measurement and structural models. Interpretational confounding produces marked changes in the estimates of the measurement parameters when alternative structural models are estimated. They also noted that when a latent variable is unidimensional, the measurement parameter estimates for that latent variable should change trivially, if at all, between the measurement and structural model estimations (J. C. Anderson & Gerbing, 1988, p. 418).

As a result, if X and Z are each unidimensional—that is, their indicators have only one underlying construct each (Aker & Bagozzi, 1979; J. C. Anderson & Gerbing, 1988; Burt, 1973; Hattie, 1985; Jöreskog, 1970, 1971; McDonald, 1981)—estimates of the parameters appearing in Equations 1 and 3 are available in a measurement model that contains X and Z but excludes XX and XZ. To explain this result, the measurement parameters of a unidimensional latent variable are by definition

⁴ The number of required constraint equations specifications in LISREL 8 is approximately equal to the number of additional variable specifications required by COSAN. Each interaction latent variable requires the specification of $2pq$ equations, and each quadratic latent variable requires $p(p + 1)$ equations, where p and q are the number of indicators for the linear variables comprising the interaction or quadratic latent variable.

Table 2
Artificial Data Set Sample Variance-Covariance Matrix

Quadratic term model						
Variable	x_1	x_2	x_1x_1	x_1x_2	x_2x_2	y
x_1	1.272					
x_2	0.605	0.826				
x_1x_1	0.202	0.105	3.614			
x_1x_2	0.208	0.131	1.687	1.553		
x_2x_2	0.162	0.141	0.860	1.175	1.731	
y	0.186	0.112	-1.317	-0.729	-0.443	0.898

Interaction term model									
Variable	x_1	x_2	z_1	z_2	x_1z_1	x_1z_2	x_2z_1	z_2z_2	y
x_1	2.376								
x_2	1.234	1.492							
z_1	0.319	0.139	2.129						
z_2	0.206	0.068	1.174	1.497					
x_1z_1	0.106	0.004	0.179	0.199	5.130				
x_1z_2	-0.244	-0.121	0.174	0.187	2.830	3.927			
x_2z_1	-0.003	0.074	0.166	0.095	2.516	1.326	2.885		
x_2z_2	-0.125	-0.161	0.085	0.096	1.333	1.908	1.572	2.153	
y	-0.232	-0.188	0.595	0.453	2.688	2.009	1.396	1.040	2.324

unaffected by the presence or absence of other latent variables in a structural model. Consequently, other latent variables can be added or deleted from a measurement or structural model containing a unidimensional latent variable with no effect on the measurement parameter estimates for that latent variable. Thus, if X and Z are unidimensional, the parameter estimates for Equation 1 or 3 could be obtained from a measurement model that excludes XX and XZ. Similarly, the addition of XX, XZ, or both to a structural model does not affect the measurement parameter estimates of X or Z in this structural model if X and Z are unidimensional.

Thus, Equations 1 and 3 loadings and error variances for product indicators such as x_1z_1 and x_1x_1 could be calculated using parameter estimates from a measurement model that excludes XX and XZ. Because these measurement parameter estimates should change trivially, if at all, between the measurement and structural model estimations (J. C. Anderson & Gerbing, 1988), these calculated loadings and error variances could then be used as fixed values (constants) in a structural equation model containing the interaction and quadratic latent variables XX and XZ.

In particular, for indicators in mean deviation form and under the Kenny and Judd (1984) normality assumptions stated in conjunction with Equation 1, Equations 1 and 3 can be simplified as follows:

$$\text{Var}(xz) = a^2\text{Var}(XZ) + \text{Var}(b). \tag{5}$$

In Equation 5, $\text{Var}(xz)$ is the variance of the indicator xz , $\text{Var}(XZ)$ is the variance of the latent variable XZ , and $a = \lambda_x\lambda_z$. $\text{Var}(b)$, the error variance for xz , is given by

$$\begin{aligned} \text{Var}(b) = & K\lambda_x^2\text{Var}(X)\text{Var}(\epsilon_z) + K\lambda_z^2\text{Var}(Z)\text{Var}(\epsilon_x) \\ & + K\text{Var}(\epsilon_x)\text{Var}(\epsilon_z), \end{aligned}$$

($K = 2$ if $x = z$, $K = 1$ otherwise). Then, if X and Z are each unidimensional, values for the loading a and the error variance for xz , $\text{Var}(b)$, can be calculated using measurement model estimates for λ_x , λ_z , $\text{Var}(X)$, $\text{Var}(Z)$, $\text{Var}(\epsilon_x)$, and $\text{Var}(\epsilon_z)$. The loading and error variance of xz can subsequently be specified using these calculated values as fixed (constant) terms in a structural model involving XX, XZ, or both, instead of variables to be estimated as the Kenny and Judd technique requires.

Consequently, the Figure 1 structural model could be estimated by setting the loadings and error variances for the product indicators equal to constants that are calculated using Equation 5 and parameter estimates from a linear-latent-variable-only measurement model involving only X, Z, and Y.

To illustrate this technique, the results of two tests of the technique's recovery of known parameters are presented.

Examples

Artificial Data Sets

Method. The proposed technique was used to recover known parameters in two artificial data sets. Using a normal random number generator, two sets of 500 cases were created. One set of 500 cases contained values based on the Table 1 population characteristics for x_1 , x_2 , and Y in the Figure 2 quadratic model. The other set of 500 cases

contained values based on the Table 1 population characteristics for x_1 , x_2 , z_1 , z_2 , and Y in the Figure 3 interaction model. These data sets were generated to meet the Kenny and Judd (1984) normality and mean deviation assumptions stated in conjunction with Equation 1.

The covariance matrices for these two data sets are shown in Table 2. The Figure 2 structural model was specified by first estimating the parameters in a linear-latent-variable-only measurement model that excluded XX. Next, the Equation 5 loadings (a s) and error variances [$\text{Var}(b)$ s] for the product indicators in Figure 2 were calculated using parameter estimates from this linear-latent-variable-only measurement model. Finally, the Figure 2 structural model was estimated with the loadings and error variances of the nonlinear latent variables fixed at their respective a and $\text{Var}(b)$ values.

The linear-latent-variable-only measurement model associated with the Figure 2 model was estimated using LISREL 7 and maximum likelihood (ML). This produced the Table 3 estimates for the λ bdas, $\text{Var}(\epsilon)$ s, and $\text{Var}(X)$ to be used in calculating the Equation 5 values for the product indicators of XX. Next, the Equation 5 values for $a_{x_1x_1}$, $a_{x_1x_2}$, $a_{x_2x_2}$, $\text{Var}(b_{x_1x_1})$, $\text{Var}(b_{x_1x_2})$, and $\text{Var}(b_{x_2x_2})$ were computed (see Figure 2 for the equations and Table 3 for the values and example calculations). Then, the structural model shown in Figure 2 was specified by fixing the loading and error variance for each product indicator to the appropriate a and $\text{Var}(b)$ values computed in the previous step. The results of the Figure 2 structural model estimation using LISREL 7 and ML are shown in Table 4.

This process was repeated for the interaction model shown in Figure 3, and the results shown in Tables 3 and 4 were obtained.

To obtain a basis for comparing the efficacy of the proposed technique, Kenny and Judd (1984) and Hayduk (1987) estimates were also

Table 3
Artificial Data Set Sample Measurement Model Parameter Estimates

Parameter	Variance	Value
Quadratic term model		
X	1.009	
ϵ_{x1}	0.262	
ϵ_{x2}	0.463	
λ_{x1}		1.000
λ_{x2}		0.599
Interaction term model		
X	2.223	
Z	1.604	
$\psi_{x,z}$	0.237	
ϵ_{x1}	0.152	
ϵ_{x2}	0.807	
ϵ_{z1}	0.524	
ϵ_{z2}	0.637	
λ_{x1}		1.000
λ_{x2}		0.554
λ_{z1}		1.000
λ_{z2}		0.731

Note. Equation 5 values: $a_{1,1} = 1.000$, $\text{Var}(b_{1,1}) = 1.198$; $a_{1,2} = 0.599$, $\text{Var}(b_{1,2}) = 0.684$; and $a_{2,2} = 0.359$, $\text{Var}(b_{2,2}) = 1.101$. For example, $a_{2,2} = \lambda_{x2}^2 = .599^2 = .359$, $\text{Var}(b_{2,2}) = 4\lambda_{x2}^2\text{Var}(X)\text{Var}(\epsilon_{z2}) + 2\text{Var}(\epsilon_{z2})^2 = 4(0.599)^2(1.009)(0.463) + 2(.463)^2 = 1.101$. Equation 5 values: $a_{1,1} = 1.000$, $\text{Var}(b_{1,1}) = 1.492$; $a_{1,2} = 0.731$, $\text{Var}(b_{1,2}) = 1.646$; $a_{2,1} = 0.554$, $\text{Var}(b_{2,1}) = 2.077$; and $a_{2,2} = 0.406$, $\text{Var}(b_{2,2}) = 1.644$. For example, $a_{2,2} = \lambda_{x2}\lambda_{z2} = 0.554 * 0.731 = 0.406$, $\text{Var}(b_{2,2}) = \lambda_{x2}^2\text{Var}(X)\text{Var}(\epsilon_{z2}) + \lambda_{z2}^2\text{Var}(Z)\text{Var}(\epsilon_{x2}) + \text{Var}(\epsilon_{z2})\text{Var}(\epsilon_{x2}) = 0.554^2(2.223)(0.637) + 0.731^2(1.604)(0.807) + (0.807)(0.637) = 1.644$.

Table 4
Structural Model Parameter Estimates

Parameter	Approach							
	Population		Kenny and Judd (1984)		Hayduk (1987)		Proposed	
	Variance	Value	Variance	Value	Variance	Value	Variance	Value
Quadratic term model								
X	1.00		1.050		1.080		1.009	
ϵ_{x1}	0.15		0.243		0.243		0.264	
ϵ_{x2}	0.55		0.516		0.524		0.463	
E_Y	0.20		0.102		0.098		0.235	
λ_{x1}		1.00		1.000		1.000		1.000
λ_{x2}		0.60		0.585		0.592		0.600
$\gamma_{Y,X}$		0.25		0.275		0.274		0.290
$\gamma_{Y,XX}$		-0.50		-0.573		-0.570		-0.494
MSE ^a -all parameters				0.003		0.004		0.003
MSE ^a - γ s				0.003		0.003		0.001
Interaction term model								
X	2.15		2.277		2.368		2.223	
Z	1.60		1.643		1.605		1.604	
$\psi_{X,Z}$	0.20		0.336		0.161		0.237	
ϵ_{x1}	0.36		0.122		0.144		0.327	
ϵ_{x2}	0.81		0.710		0.769		0.747	
ϵ_{z1}	0.49		0.450		0.462		0.538	
ϵ_{z2}	0.64		0.678		0.692		0.638	
E_Y	0.16		0.224		0.186		0.353	
λ_{x1}		1.00		1.000		1.000		1.000
λ_{x2}		0.60		0.543		0.537		0.599
λ_{z1}		1.00		1.000		1.000		1.000
λ_{z2}		0.70		0.722		0.720		0.737
$\gamma_{Y,X}$		-0.15		-0.145		-0.140		-0.132
$\gamma_{Y,XZ}$		0.70		0.695		0.780		0.666
$\gamma_{Y,Z}$		0.35		0.317		0.320		0.318
MSE ^a -all parameters				0.008		0.007		0.004
MSE ^a - γ s				0.000		0.002		0.001
Overall								
MSE ^a -all parameters				0.006		0.007		0.003
MSE ^a - γ s				0.001		0.003		0.001

^a Mean squared deviations from the population parameters.

generated. These estimates used the Figures 2 and 3 models. The Kenny and Judd estimates were produced using COSAN and generalized least squares (GLS), and the Hayduk estimates used LISREL 7 and ML. The results are shown in Table 4.

Results. The three estimation techniques produced essentially equivalent parameter estimates. The estimates were within a few points of the population values and each other. The squared average deviations from the population values (MSEs in Table 4) produced by each technique were also within a few points of each other. For the quadratic model, the overall MSE values for the three techniques (MSE-all parameters of the quadratic term model in Table 4) were nearly identical. The MSE for the quadratic effect coefficients produced by the proposed technique (MSE- γ s) was slightly smaller than it was for the Hayduk (1987) and Kenny and Judd (1984) techniques. For the interaction model in the table, the all-parameter MSEs were also within a few points of each other. However, the all-param-

eter MSEs were slightly larger than they were for the quadratic model, the effect coefficient MSEs were smaller, and the Kenny and Judd technique produced the smallest effect coefficient MSE. Combining the parameter estimates for the two models (see Overall in Table 4), the proposed technique produced MSEs that were the same or slightly smaller than the Hayduk and Kenny and Judd techniques.

To illustrate the use of the proposed technique, a field survey data analysis involving nonlinear latent variables is presented.

A Field Survey

Method. As part of a larger study of a social exchange view of long-term buyer-seller relationships involving business firms, data were gathered from key informants in retailing firms concerning their loyalty to their primary economic exchange partner, their primary wholesaler; their satisfaction with that economic exchange partner; and the attrac-

tiveness of the best alternative wholesaler. Relationship satisfaction (SAT) and alternative attractiveness (ALT) were hypothesized to affect loyalty (LOY; see Ping, 1993; Rusbult, Zembrodt, & Gunn, 1982).

Because this is an illustration of the use of the proposed estimation technique, the study is simply sketched. SAT, ALT, and LOY were measured using multiple-item measures. Survey responses were used to create indicators of the independent variables (i.e., SAT and ALT) in mean deviation form. The responses were then used to assess the unidimensionality of SAT, ALT, and LOY. They were also used to gauge the normality of the linear indicators using the skewness and kurtosis tests (see, e.g., Mardia, 1970) in LISREL 7's PRELIS.

Values for the product indicators were created for each survey response by forming all unique products of the values of the appropriate indicators of the linear latent variables, then appending these products to the response (see comments regarding the formation of these indicators at the bottom of Table 6). Next, the linear-latent-variable-only measurement model for the Figure 4 model (i.e., with SAT, ALT, and LOY only) was estimated. This was accomplished using the Table 5 variance-covariance matrix, ML, and LISREL 7. The resulting measurement parameter estimates for the Equation 5 *as* and *Var(b)s* are shown in Table 6.

The structural equation was then estimated by calculating the Figure 4 product indicator loadings and error variances [*as* and *Var(b)s*], using the Table 6 measurement model estimates and Equation 5 (see Table 6).⁵ Then, the loadings and error variances for the product indicators were fixed at these calculated values in the structural model, and the structural equation estimates shown in Table 7 were produced using LISREL 7 and ML. Table 7 also shows the ML estimates using the Kenny and Judd (1984) technique for comparison.

Results. The estimates produced by the Kenny and Judd (1984) technique and the proposed technique were again similar. Although some were higher and some were lower, the calculated *as* and *Var(b)s* produced by the proposed technique were within a few points of the Kenny and Judd estimates for the

loadings and error variances of the product indicators. Similarly, the structural effect coefficients (lambdas) for the two techniques were comparable.

Discussion

As the results in Tables 6 and 7 show, the measurement parameter estimates for the unidimensional SAT and ALT variables changed trivially between the linear-latent-variable-only measurement model and the Figure 4 structural model that contained the linear and nonlinear latent variables. Procedures for obtaining unidimensionality are suggested in articles by J. C. Anderson and Gerbing (1982; Gerbing & Anderson, 1988) and Jöreskog (1993). Although there is no agreement on the detailed steps, the process of obtaining unidimensionality must balance concern for the content validity of a measure with its consistency. In the field survey example, the estimation of single construct measurement models (Jöreskog, 1993) with a target comparative fit index (Bentler, 1990) of 0.99 produced the desired trivial difference in measurement parameters between the measurement and structural models.⁶

Had more than a trivial change been observed in the measurement parameters for the linear latent variables between the measurement and structural models (i.e., differences in the second decimal place), measurement parameter estimates for the linear latent variables taken from the structural model could have been used to recompute the *as* and *Var(b)s* and to re-estimate the structural model. As a result, an iterative process could be used with measurement parameter estimates for the linear latent variables from the previous structural model to recompute the *as* and *Var(b)s*, and thereby "converge" to the desired trivial change between structural model estimates of the measurement parameters for the linear latent variables.⁷

The assumption that the error terms for linear indicators are independent can be relaxed. In Equation 5, the expression for *Var(b)* would be changed by an additional covariance term (available in the measurement model) as follows,

$$\begin{aligned} \text{Var}(b) = & K\lambda_x^2 \text{Var}(X) \text{Var}(\epsilon_z) + K\lambda_z^2 \text{Var}(Z) \text{Var}(\epsilon_x) \\ & + K \text{Var}(\epsilon_x) \text{Var}(\epsilon_z) + (2 - K)\lambda_x\lambda_z \text{Cov}(X, Z) \text{Cov}(\epsilon_x, \epsilon_z), \end{aligned}$$

(K = 2 if x = z, K = 1 otherwise).

Limitations

Just as in the Hayduk (1987) and Kenny and Judd (1984) techniques, the assumption of normality in the linear indicators can-

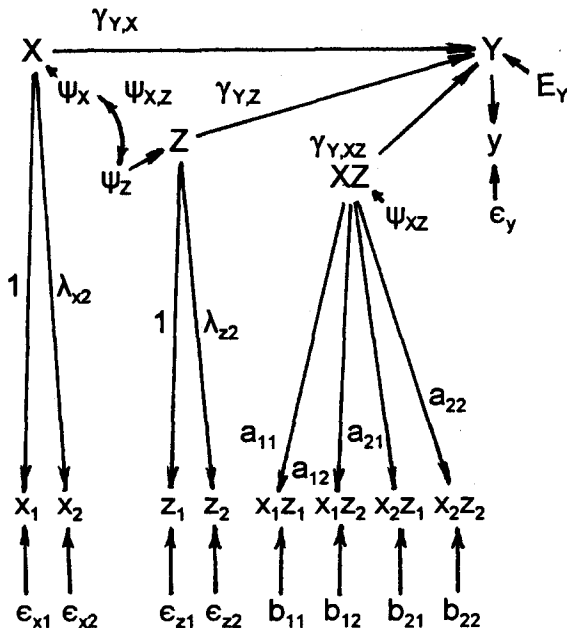


Figure 3. An interaction model using the proposed approach. $a_{11} = \lambda_{x1}\lambda_{z1} = 1$, $a_{12} = \lambda_{x1}\lambda_{z2} = \lambda_{z2}$; $a_{21} = \lambda_{x2}\lambda_{z1} = \lambda_{x2}$, $a_{22} = \lambda_{x2}\lambda_{z2}$; $b_{11} = \lambda_{x1}X\epsilon_{x1} + \lambda_{z1}Z\epsilon_{z1} + \epsilon_{x1}\epsilon_{z1}$; $b_{12} = \lambda_{x1}X\epsilon_{x2} + \lambda_{z2}Z\epsilon_{z1} + \epsilon_{x1}\epsilon_{z2}$; $b_{21} = \lambda_{x2}X\epsilon_{z1} + \lambda_{z1}Z\epsilon_{x2} + \epsilon_{x2}\epsilon_{z1}$; $b_{22} = \lambda_{x2}X\epsilon_{z2} + \lambda_{z2}Z\epsilon_{z2} + \epsilon_{x2}\epsilon_{z2}$.

⁵ See the representative calculations in Table 3. A computer program written in BASIC is available from the author to calculate *as* and *Var(b)s*.

⁶ For each linear latent variable, a single construct measurement model (Jöreskog, 1993) was re-estimated until a target comparative fit index (Bentler, 1990) value of 0.99 was attained by serially deleting items that did not appear to degrade construct validity.

⁷ A reviewer suggested this procedure to deal with slight differences between the measurement parameter estimates from the linear-latent-variable-only measurement model and their estimates in the structural model. The effectiveness of this procedure for larger measurement parameter differences (i.e., in the first decimal place) is unknown.

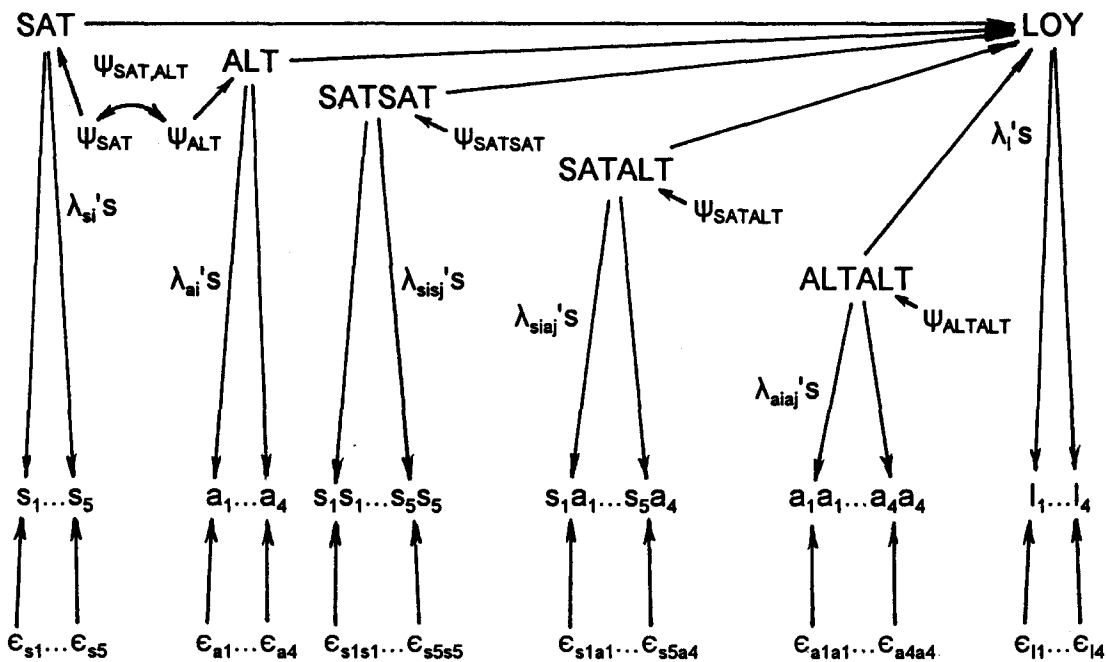


Figure 4. A field survey model using the proposed approach. To avoid notation such as $a_{ai,nj}$, the a s and b s are denoted by lambda and epsilon respectively, for example, $\lambda_{ai,sj} = a_{ai,sj}$ and $\epsilon_{altl} = b_{altl}$. SAT = relationship satisfaction; ALT = alternative attractiveness; LOY = loyalty to primary wholesaler.

not be relaxed. The derivation of the a and $\text{Var}(b)$ terms is based on this assumption. Bollen (1989, pp. 418–424) discussed appropriate normality tests involving indicator skewness and kurtosis. EQS and LISREL's PRELIS include several of these tests. However, for typical sample sizes used in structural equation analysis, even small deviations from normality are likely to be statistically significant (Bentler, 1989). In addition, there is little guidance for determining when statistical nonnormality becomes practical nonnormality (Bentler, 1989). As a result, whereas the survey items were judged to be “not nonnormal,” several items were statistically nonnormal using standard skewness and kurtosis tests (although the coefficients were not unreasonably large). In addition,

the Mardia (1970) coefficient of multivariate nonnormality was significant (although not excessively so).

The robustness of the proposed, Hayduk (1987), and Kenny and Judd (1984) techniques to departures from normality is not known, and unreasonable departures from multivariate normality should be remedied. Bollen (1989) suggested transformation of the data (p. 425; see Neter, Wasserman, & Kunter, 1988, for alternatives to the \log_e transformation), and Bentler (1989) discussed the deletion of cases that contribute to nonnormality (p. 228).

In the proposed, Hayduk (1987), and Kenny and Judd (1984) techniques, the product indicators are not normally dis-

Table 5
Field Data Set Variance-Covariance Matrix

Observed variable	s_1	s_2	s_3	s_4	a_5	a_1	a_2	a_3	a_4	l_1	l_2	l_3	l_4
s_1	.49												
s_2	.36	.53											
s_3	.41	.46	.62										
s_4	.37	.40	.44	.52									
s_5	.37	.43	.48	.43	.56								
a_1	-.28	-.34	-.35	-.29	-.31	1.00							
a_2	-.25	-.30	-.32	-.24	-.31	.71	.94						
a_3	-.26	-.33	-.38	-.32	-.34	.79	.76	.92					
a_4	-.21	-.29	-.32	-.22	-.29	.59	.61	.66	.76				
l_1	-.06	-.03	.04	-.05	-.05	.03	.08	.05	.05	.70			
l_2	.00	.02	.03	.04	.02	.04	.07	.02	.03	.54	.68		
l_3	.05	.11	.14	.11	.09	-.07	-.07	-.12	-.09	.35	.42	.88	
l_4	-.13	-.05	-.14	-.08	-.14	-.02	.06	.00	.05	.39	.41	.30	.96

Table 6
Field Data Set Measurement Model Parameter Estimates

Parameter	Variance	Parameter	Value
SAT	0.519	ϵ_{14}	0.680
ALT	0.850	λ_{a1}	0.793
$\psi_{x,z}$	-0.372	λ_{a2}	0.883
ϵ_{a1}	0.167	λ_{a3}	1.000
ϵ_{a2}	0.132	λ_{a4}	0.880
ϵ_{a3}	0.108	λ_{a5}	0.939
ϵ_{a4}	0.119	λ_{a1}	0.924
ϵ_{a5}	0.102	λ_{a2}	0.904
ϵ_{a1}	0.275	λ_{a3}	1.000
ϵ_{a2}	0.248	λ_{a4}	0.783
ϵ_{a3}	0.077	λ_{11}	0.876
ϵ_{a4}	0.244	λ_{12}	1.000
ϵ_{11}	0.234	λ_{13}	0.688
ϵ_{12}	0.069	λ_{14}	0.682
ϵ_{13}	0.589		

Note. Equation 5 values ($\lambda_s = as, \epsilon_s = bs$):

$\lambda_{s1,s1} = .628$ Var($\epsilon_{s1,s1}$) = .273	$\lambda_{a1,a1} = .853$ Var($\epsilon_{a1,a1}$) = .949	$\lambda_{a2,a2} = .798$ Var($\epsilon_{a2,a2}$) = .224
$\lambda_{s1,s2} = .700$ Var($\epsilon_{s1,s2}$) = .132	$\lambda_{a1,a2} = .835$ Var($\epsilon_{a1,a2}$) = .439	$\lambda_{a2,a3} = .883$ Var($\epsilon_{a2,a3}$) = .153
$\lambda_{s1,s3} = .793$ Var($\epsilon_{s1,s3}$) = .139	$\lambda_{a1,a3} = .924$ Var($\epsilon_{a1,a3}$) = .310	$\lambda_{a2,a4} = .691$ Var($\epsilon_{a2,a4}$) = .199
$\lambda_{s1,s4} = .697$ Var($\epsilon_{s1,s4}$) = .125	$\lambda_{a1,a4} = .723$ Var($\epsilon_{a1,a4}$) = .387	$\lambda_{a3,a1} = .924$ Var($\epsilon_{a3,a1}$) = .250
$\lambda_{s1,s5} = .744$ Var($\epsilon_{s1,s5}$) = .126	$\lambda_{a2,a2} = .817$ Var($\epsilon_{a2,a2}$) = .812	$\lambda_{a3,a2} = .904$ Var($\epsilon_{a3,a2}$) = .230
$\lambda_{s2,s2} = .779$ Var($\epsilon_{s2,s2}$) = .248	$\lambda_{a2,a3} = .904$ Var($\epsilon_{a2,a3}$) = .283	$\lambda_{a3,a3} = 1.00$ Var($\epsilon_{a3,a3}$) = .140
$\lambda_{s2,s3} = .883$ Var($\epsilon_{s2,s3}$) = .126	$\lambda_{a2,a4} = .707$ Var($\epsilon_{a2,a4}$) = .359	$\lambda_{a3,a4} = .783$ Var($\epsilon_{a3,a4}$) = .209
$\lambda_{s2,s4} = .777$ Var($\epsilon_{s2,s4}$) = .116	$\lambda_{a3,a3} = 1.00$ Var($\epsilon_{a3,a3}$) = .273	$\lambda_{a4,a1} = .813$ Var($\epsilon_{a4,a1}$) = .229
$\lambda_{s2,s5} = .829$ Var($\epsilon_{s2,s5}$) = .115	$\lambda_{a3,a4} = .783$ Var($\epsilon_{a3,a4}$) = .266	$\lambda_{a4,a2} = .795$ Var($\epsilon_{a4,a2}$) = .211
$\lambda_{s3,s3} = 1.00$ Var($\epsilon_{s3,s3}$) = .247	$\lambda_{a4,a4} = .613$ Var($\epsilon_{a4,a4}$) = .627	$\lambda_{a4,a3} = .880$ Var($\epsilon_{a4,a3}$) = .141
$\lambda_{s3,s4} = .880$ Var($\epsilon_{s3,s4}$) = .118	$\lambda_{a1,a1} = .732$ Var($\epsilon_{a1,a1}$) = .256	$\lambda_{a4,a4} = .689$ Var($\epsilon_{a4,a4}$) = .189
$\lambda_{s3,s5} = .939$ Var($\epsilon_{s3,s5}$) = .113	$\lambda_{a1,a2} = .716$ Var($\epsilon_{a1,a2}$) = .238	$\lambda_{a5,a1} = .867$ Var($\epsilon_{a5,a1}$) = .227
$\lambda_{s4,s4} = .774$ Var($\epsilon_{s4,s4}$) = .219	$\lambda_{a1,a3} = .793$ Var($\epsilon_{a1,a3}$) = .179	$\lambda_{a5,a2} = .848$ Var($\epsilon_{a5,a2}$) = .209
$\lambda_{s4,s5} = .826$ Var($\epsilon_{s4,s5}$) = .107	$\lambda_{a1,a4} = .620$ Var($\epsilon_{a1,a4}$) = .207	$\lambda_{a5,a3} = .939$ Var($\epsilon_{a5,a3}$) = .129
$\lambda_{s5,s5} = .881$ Var($\epsilon_{s5,s5}$) = .207	$\lambda_{a2,a1} = .815$ Var($\epsilon_{a2,a1}$) = .243	$\lambda_{a5,a4} = .735$ Var($\epsilon_{a5,a4}$) = .189

Because relationship satisfaction (SAT) and alternative attractiveness (ALT) have 5 and 4 indicators respectively, SAT*SAT has 15 product indicators [= $p * (p + 1)/2$, where p is the number of indicators], one for each unique product of the indicators of SAT, and that many sets of Equation 3 as and $Var(bs)$ (one set for each product indicator). Similarly, ALT*ALT has 10 sets of as and $Var(bs)$ and SAT*ALT has 20 (= $p*q$, where p and q are the number of indicators of SAT and ALT respectively).

tributed. This means that the customary ML and GLS estimators are formally inappropriate for these techniques because they assume multivariate normality.

This presents several apparent difficulties in using these techniques: Structural model parameter estimates and the fit and significance statistics may be incorrect. However, on the basis of available evidence (e.g., T. W. Anderson & Amemiya, 1985, 1986; Boomsma, 1983; Browne, 1987; Harlow, 1985; Sharma, Durvasula, & Dillon, 1989; Tanaka, 1984), ML and GLS parameter estimates are robust against departures from normality (Bollen, 1989; Jöreskog & Sörbom, 1989). The results of the present study support this: Figures 2 and 3 models were not multivariate normal (because the product indicators are not normally distributed), yet the proposed, Hayduk (1987), and Kenny and Judd (1984) techniques reproduced the population parameters quite well using ML and GLS estimates.⁸

For model fit and significance statistics, however, these estimation techniques should be used with caution (Bentler, 1989; Bollen, 1989; Hu, Bentler, & Kano, 1992; Jöreskog & Sörbom, 1989).⁹ Additional estimators that are less dependent on distributional assumptions should be used with these techniques to

determine model fit and significance. EQS and LISREL provide asymptotic distribution-free estimation (Browne, 1982, 1984).¹⁰ EQS also provides linearized distribution-free estimation (Bentler, 1983) and Robust statistics (Satorra & Bentler, 1988). For large models, fit indices (see Bollen & Long, 1993) may be appropriate (Hayduk, 1987; Kenny & Judd, 1984). This is obviously an area where additional work is needed.

⁸ Kenny and Judd (1984) and Hayduk (1987) reported similar results.

⁹ Results from recent investigations (see Jaccard & Wan, 1995; Ping, in press) suggest that model fit and significance statistics from ML and possibly GLS estimators are robust to the addition of a few nonlinear indicators (e.g., x_1z_1) involving linear indicators (e.g., x_1 and z_1) that are normally distributed. However, the robustness of model fit and significance statistics from these estimators to the addition of many nonlinear indicators (i.e., over four) or nonlinear indicators comprised of nonnormal linear indicators (typical of survey data) is unknown.

¹⁰ As Aiken and West (1991) warn, and other authors suggest (see Hu, Bentler, & Kano, 1992; Jaccard & Wan, 1995), results from asymptotic distribution-free estimation with less than very large sample sizes also seem to require cautious interpretation.

Table 7
Field Data Set Structural Parameter Estimates

Variance			Variance			Variance-value		
Parameter	Proposed	COSAN	Parameter	Proposed	COSAN	Parameter	Proposed (t value) ^a	COSAN (t value) ^a
SAT	0.519	0.554	$\lambda_{s4,s5}$	0.826	0.839	ϵ_{s3a2}	0.230	0.219
ALT	0.851	0.892	$\lambda_{s5,s5}$	0.881	0.897	ϵ_{s3a3}	0.140	0.152
λ_{s1}	0.793	0.741	$\lambda_{a1,a1}$	0.853	0.866	ϵ_{s3a4}	0.209	0.187
λ_{s2}	0.883	0.875	$\lambda_{a1,a2}$	0.835	0.821	ϵ_{s4a1}	0.229	0.210
λ_{s3}	1.000	1.000	$\lambda_{a1,a3}$	0.924	0.911	ϵ_{s4a2}	0.211	0.200
λ_{s4}	0.880	0.872	$\lambda_{a1,a4}$	0.723	0.734	ϵ_{s4a3}	0.141	0.149
λ_{s5}	0.940	0.931	$\lambda_{a2,a2}$	0.817	0.824	ϵ_{s4a4}	0.189	0.169
λ_{a1}	0.924	0.930	$\lambda_{a2,a3}$	0.904	0.906	ϵ_{s5a1}	0.227	0.238
λ_{a2}	0.905	0.890	$\lambda_{a2,a4}$	0.707	0.692	ϵ_{s5a2}	0.209	0.217
λ_{a3}	1.000	1.000	$\lambda_{a3,a3}$	1.000	1.000	ϵ_{s5a3}	0.129	0.109
λ_{a4}	0.783	0.774	$\lambda_{a3,a4}$	0.783	0.796	ϵ_{s5a4}	0.189	0.201
λ_{11}	0.879	0.860	$\lambda_{a4,a4}$	0.613	0.600	ϵ_{s1s1}	0.273	0.251
λ_{12}	1.000	1.000	$\lambda_{s1,a1}$	0.732	0.739	ϵ_{s1s2}	0.132	0.115
λ_{13}	0.692	0.708	$\lambda_{s1,a2}$	0.716	0.731	ϵ_{s1s3}	0.139	0.147
λ_{14}	0.686	0.671	$\lambda_{s1,a3}$	0.793	0.781	ϵ_{s1s4}	0.125	0.139
ϵ_{s1}	0.166	0.134	$\lambda_{s1,a4}$	0.620	0.610	ϵ_{s1s5}	0.126	0.141
ϵ_{s2}	0.133	0.101	$\lambda_{s2,a1}$	0.815	0.824	ϵ_{s2s2}	0.248	0.250
ϵ_{s3}	0.108	0.129	$\lambda_{s2,a2}$	0.798	0.806	ϵ_{s2s3}	0.126	0.114
ϵ_{s4}	0.119	0.101	$\lambda_{s2,a3}$	0.883	0.898	ϵ_{s2s4}	0.116	0.134
ϵ_{s5}	0.102	0.079	$\lambda_{s2,a4}$	0.691	0.678	ϵ_{s2s5}	0.115	0.111
ϵ_{a1}	0.275	0.194	$\lambda_{s3,a1}$	0.924	0.931	ϵ_{s3s3}	0.247	0.237
ϵ_{a2}	0.248	0.262	$\lambda_{s3,a2}$	0.904	0.900	ϵ_{s3s4}	0.118	0.136
ϵ_{a3}	0.077	0.124	$\lambda_{s3,a3}$	1.000	1.000	ϵ_{s3s5}	0.113	0.116
ϵ_{a4}	0.244	0.219	$\lambda_{s3,a4}$	0.783	0.772	ϵ_{s4a4}	0.219	0.202
ϵ_{11}	0.232	0.247	$\lambda_{s4,a1}$	0.813	0.827	ϵ_{s4a5}	0.107	0.115
ϵ_{12}	0.072	0.061	$\lambda_{s4,a2}$	0.795	0.808	ϵ_{s5s5}	0.207	0.217
ϵ_{13}	0.587	0.569	$\lambda_{s4,a3}$	0.880	0.871	ϵ_{a1a1}	0.949	1.001
ϵ_{14}	0.677	0.702	$\lambda_{s4,a4}$	0.689	0.694	ϵ_{a1a2}	0.439	0.479
$\lambda_{s1,s1}$	0.621	0.615	$\lambda_{s5,a1}$	0.867	0.853	ϵ_{a1a3}	0.310	0.321
$\lambda_{s1,s2}$	0.700	0.710	$\lambda_{s5,a2}$	0.848	0.840	ϵ_{a1a4}	0.387	0.374
$\lambda_{s1,s3}$	0.793	0.781	$\lambda_{s5,a3}$	0.939	0.930	ϵ_{a2a2}	0.812	0.813
$\lambda_{s1,s4}$	0.697	0.705	$\lambda_{s5,a4}$	0.735	0.748	ϵ_{a2a3}	0.283	0.267
$\lambda_{s1,s5}$	0.744	0.759	ϵ_{s1a1}	0.256	0.271	ϵ_{a2a4}	0.359	0.349
$\lambda_{s2,s2}$	0.779	0.769	ϵ_{s1a2}	0.238	0.227	ϵ_{a3a3}	0.273	0.237
$\lambda_{s2,s3}$	0.883	0.880	ϵ_{s1a3}	0.179	0.161	ϵ_{a3a4}	0.266	0.232
$\lambda_{s2,s4}$	0.777	0.798	ϵ_{s1a4}	0.207	0.216	ϵ_{a4a4}	0.627	0.639
$\lambda_{s2,s5}$	0.829	0.841	ϵ_{s2a1}	0.243	0.254	$\gamma_{LOY,SAT}$	0.031 (0.3)	0.020 (0.2)
$\lambda_{s3,s3}$	1.000	1.000	ϵ_{s2a2}	0.224	0.256	$\gamma_{LOY,ALT}$	0.097 (1.4)	0.099 (1.4)
$\lambda_{s3,s4}$	0.880	0.871	ϵ_{s2a3}	0.153	0.171	$\gamma_{LOY,SAT*ALT}$	0.384 (6.0)	0.379 (5.9)
$\lambda_{s3,s5}$	0.939	0.951	ϵ_{s2a4}	0.199	0.176	$\gamma_{LOY,ALT*ALT}$	0.127 (3.1)	0.119 (2.9)
$\lambda_{s4,s4}$	0.774	0.763	ϵ_{s3a1}	0.250	0.261	$\gamma_{LOY,SAT*SAT}$	0.153 (2.3)	0.161 (2.4)

Note. LOY = loyalty (to primary wholesaler); ALT = alternative attractiveness (to the best wholesaler); SAT = relationship satisfaction (with the primary wholesaler).
^a Approximate.

Finally, mean deviation form for the indicators cannot be relaxed. The derivation of a and $Var(b)$ terms were based on this assumption. Further, mean deviation form is recommended to improve the interpretability of the linear effect coefficients in regression (see Aiken & West, 1991; Jaccard et al., 1990).

Conclusion

The article has proposed an alternative to the Hayduk (1987) and Kenny and Judd (1984) techniques for estimating structural equation models with interaction or quadratic latent variables. The proposed technique is limited to indicators that are in mean deviation form and multivariate normal. In addition,

the linear latent variables are assumed to be unidimensional, so measurement model parameter estimates can be used in the structural model as constants. An iterative procedure is suggested to correct for slight differences in the measurement parameter estimates of linear latent variables between the measurement and structural models. The efficacy of the proposed technique is suggested by recovering known parameters in artificial data sets and by producing estimates for field survey data that are similar to Kenny and Judd estimates.

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