ML Poisson Toy

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QII - Poisson Likelihood Toy Example

Poisson

- Suppose we have N draws from a Poisson distribution with parameter λ

$$f(y_i|\lambda) = \frac{e^{-\lambda}\lambda^{y_i}}{y_i!}; \lambda > 0 \tag{1}$$

- Likelihood is then

$$L = \prod_{i=1}^{N} \frac{e^{-\lambda}\lambda^{y_i}}{y_i!} \tag{2}$$

- or the log likelihood is

$$\ln L = \sum_{i=1}^{N} [-\lambda + y_i \ln \lambda - \ln y!] \tag{3}$$

- This is a very well behaved likelihood which is easy to maximize.

Calculus

- First order condition is very easy to get:
-
$$\frac{d \ln L}{d\lambda} = \sum_{i=1}^{N} \left(\frac{y_i}{\lambda} - 1\right) = \mathbf{0} \tag{4}$$
- This yields $\hat{\lambda} = \frac{\sum_{i=1}^{N} y_i}{N}$

Curvature - second derivative

- The second derivative (only one parameter, so is derivative, not partial derivative)
-
$$\frac{d^2 \ln L}{d\lambda^2} = -\sum_{i=1}^{N} \frac{y_i}{\lambda^2} \tag{5}$$
$$= \frac{-1}{\lambda^2} \sum_{i=1}^{N} y_i \tag{6}$$
- Thus the critical point we found is a max
- Everything in the sum is non-negative and something is positive
- And the log likelihood is globally concave

- At the ml estimate $\hat{\lambda} = \frac{\sum_{i=1}^{N} y_i}{N}$
- So evaluating the second derivative at $\hat{\lambda}$ we have
- $$\frac{d^2 \ln L}{d\lambda^2} = \frac{-N}{\hat{\lambda}} \quad (7)$$
- Note that as we get a bigger N, the second derivative, evaluated at the ml estimate, $\hat{\lambda}$ becomes more negative
- As $\hat{\lambda} \rightarrow 0$ the second derivative becomes more negative
- We will get back to that shortly