Exercise 2
QuantII
Due: Feb. 4

1. The following data show the number of appointments to the United States Supreme Court from 1961 through 1992, by presidential term. Given that these are events which occur over a fixed interval of time, it is not unreasonable to suppose that they follow a poisson distribution.

The poisson distribution is

$$f(x_i, \lambda) \sim \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

The likelihood is

$$L(x_1, x_2, \ldots, x_8|\lambda) = \frac{e^{-8\lambda}\lambda^{\Sigma x_i}}{\prod x_i!}$$

(this is just algebraic transformation of how we did in class)

The data are

Years	Number Appointed
1961-64	2
1965-68	2
1969-72	4
1973-76	1
1977-80	0
1981-84	1
1985-88	3
1989-92	2

1a. Use maximum likelihood estimation, via a grid search, to estimate the parameter of the poisson distribution (by eye). To do this just have Stata graph the likelihood function and see where the maximum is.

1b. We usually maximize the log of the likelihood, for reasons to be discussed in class. Write down the log of the likelihood function and it to see where the maximum is (graphically).

1c You will observe that only part of the log likelihood is a function of λ. Thus the maximum does not depend on the stuff that does not vary with λ. Redo 1b dropping the parts of the likelihood that do not vary with λ.

1d Now find the maximum by calculus. Check the curvature by looking at the second derivative. Is this consistent with your graph.

1e. If we take the log of the whole likelihood function, and then simplify, we can see what functions of the y_i are sufficient statistics for λ (that is, if you knew some function of the y_i you would know all there is be known about λ, and you do not care how the individual y_i contributed to the function of y_i. What are the sufficient statistics for λ. (Hint: the answer is SIMPLE, and obvious from just looking at the log likelihood for the whole sample, or by looking at the derivatives.)

1f. Use the estimated parameter to estimate the probability that Bill Clinton would have 0, 1, 2, 3, 4 or 5 appointments to the Supreme Court during his first term of office. (He actually had 2 appointments: Ginsburg, 8/10/93; Breyer, 8/3/94.)

1g. You are still in graduate school in 2400, and so you now have not 8 data point but 80 (data collection was slow!). Assume that each succeeding 32 years looks exactly like the first 32 years (that is, you can use the stata command expand = 10. Repeat 1a-d for the bigger data set. What stays the same? what changes? by how much?

2. Until recently, Japan's electoral system used multimember districts with single non-transferable votes, an unusual and interesting electoral system. The following table shows the number of seats won by the LDP (the ruling party during most of this electoral system) in 6 recent elections in a single district. The district has 5 seats. We observe only the number of seats won. From this, we would like to estimate the probability that the LDP wins each seat. We must assume that the winner of each seat is independent of the winner of all other seats (a possibly dubious assumption). Since the district magnitude is fixed at 5 seats, the binomial distribution seems appropriate to this problem.

The binomial distribution is

$$f(y_i, \pi) \sim \frac{N!}{y_i!(N - y_i)!} \pi^{y_i} (1 - \pi)^{N - y_i}$$

The likelihood is

$$L(y_1, y_2, \ldots, y_6) = \frac{N!^6}{\prod y_i!(N - y_i)!} \pi^{\Sigma y_i} (1 - \pi)^{\Sigma (N - y_i)}$$

The data are

Election	LDP Seats
1	2
2	4
3	2
4	4
5	5
6	3

Repeat 1a-d for this dataset and model

2f. Use the estimate of π to calculate the probability distribution for seat outcomes from 0 to 5 LDP seats won.

3. Suppose we have a sample $x_1, \ldots, x_N$ from a population which is $N(\mu, \sigma^2$. Derive the ml estimates of mu and σ^2 (jointly).