Longdata notes: Sept. 8. 2003 - Enders, ch. 1 - Difference equations

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Enders is very clear, so I only need emphasize what is really important

- Notation: $\Delta y_t = y_t - y_{t-1}$
- Difference between structural and reduced form of a model; reduced form for y has only lagged values of y and forcing (see below) variables
- Forcing variables: what we might call exogenous. Can either be xs that are not caused by y but cause y, or errors. At present, either are the same
- Solution to reduced form: write y_t in terms of forcing process and initial conditions on y (y_0), no general lagged values of y in solution
- Notion of equilibrium ($y_t^E = y_{t+1}^E$)
- How does system get to equilibrium
- Is equilibrium stable?
- Memory of proces - role of y_0 in determining y_t when t is big
- Stability of system - does it explode as t grows?
- Solution by forward iteration from known y_0
- Solution by backward iteratio - at some point distant past doesnt matter if memory is not too long
- Hard to do for other than very simple model
- Impact multiplier - $\frac{\partial y_t}{\partial x_t}$ = first derivative of y wrt forcing variable 11. One period impact multipler - $\frac{\partial y_{t+1}}{\partial x_t}$ = first derivate of the lead of y wrt forcing variable 12. Impulse response function - graph of all impact multiplier for all leads

More general method

Details in Enders, 1-6

- For equation $y_t = a_1 y_{t-1} + a_2 y_{t-2} + x_t$ where x is an arbitrary forcing process

- First solve the homogenous equation $y_t - a_1 y_{t-1} - a_2 y_{t-2} = 0$
- if ^{S} is a solution, so is Cy^S and if y^S and y^T is a solution, so is $y^S + y^T$
- For n'th order equation, there are two general solutions and any linear combination is also a solution
- Try $y_t = \alpha^t$
- Then $\alpha^2 - a_1\alpha - a_2 = 0$
- Simple quadratic, solve for two values of α, characteristic roots by high school formula
- If roots unequal, done.
- Each root must be less than one, else unstable
- If roots equal (unlikely in practice), solutions are $\frac{a_1}{2}^t$ and $t\frac{a_1}{2}^t$. Stability if $|a_1| < 2$
- If roots imaginary, then resort to DeMoivre's Theorem to get result as on p. 27