Professor Nagler **Spring, 2001**

G53.2127 : Quantitative Political Analysis II

Practice Midterm : Spring, 2001

This exam is supposed to test your understanding of how to interpret the results of multiple regression analysis. I want to know if you understand how to test hypotheses using the multiple regression technique, and recognize its limitations under certain circumstances. You will be given all the tables that will be required to perform certain tests, and you are wellcome to use a calculator. Partial credit will be given, so try to practice making your answers as clear and as thorough as you can. You can get most of the credit for a problem even if you make some huge arithmetic mistakes, as long as I see that you are trying to use the proper solution technique. The exam is designed so that you should have plenty of time within a 60 minute session. So if you finish in 30 minutes that's fine - it doesn't mean that you have done something wrong. And if you get real confused by a problem, take your time. Good luck!

1) a) [15 points] List and explain the Gaus-Markov assumptions. You should have a verbal statement of each assumption, **and** an equation for at least four of the assumptions. Be sure to explain the meaning of any technical terms you use (i.e., "no heteroscedasticity" is *not* an explanation).

b) [5 points] State and explain the Gaus-Markov theorem.

2) [**30 points**] Consider the model: $Y_i \ = \ \beta_1 \ + \ \beta_2 * X_i \ + \ u_i$

Where Y_i is the dropout rate in school district i, and X_i is the per pupil spending in school district i. Say that you run OLS and find that:

- $\hat{\beta}_1 = 7.2$; $\sigma_{\hat{\beta}_1} = 3.10$
- $\hat{\beta}_2 =$ -1.50; $\sigma_{\hat{\beta}_2} = 0.35$
- $R^2 = 0.17$
- N = 1000

a) [10 points] Say that you define a new variable Z:

$$Z_i \ = \ Y_i \ - \ \hat{\beta}_1 \ - \ \hat{\beta}_2 * X_i \tag{1}$$

Based on the information above, what is the correlation between Z and X?

Now, assume that the Gauss-Markov assumptions are satisfied, and that u_i is normally distributed.

b) [15 points] What hypothesis about the relationship between X and Y can you test with the estimated coefficient $\hat{\beta}_2$? What is the result of your test with alpha = .05?

c) [5 points] If X changed from 30 to 10, what would you predict the change in Y to be?

3) [**50 points**] Consider the model:

$$Y_i \; = \; \beta_0 \; + \; \beta_1 * X1_i \; + \; \beta_2 * X2_i \; + \; \beta_3 * X3_i \; + \; \beta_4 * X4_i \; + \; u_i \qquad (2)$$

Say that you run OLS and find that:

- $\hat{\beta}_0 = 11.87;\ \sigma_{\hat{\beta}_0} = 11.48$
- $\hat{\beta}_1 = 1.12;\ \sigma_{\hat{\beta}_1} = 0.24$
- $\hat{\beta}_2 = 2.13;\ \sigma_{\hat{\beta}_2} = 1.24$
- $\hat{\beta}_3 = 2.01;\ \sigma_{\hat{\beta}_3} = 1.82$
- $\hat{\beta}_4 = -3.46;\ \sigma_{\hat{\beta}_4} = 2.76$

- $R^2 = 0.86$
- N = 200

Assume that the Gauss-Markov assumptions are satisfied, and that u_i is normally distributed.

a) [10 points] Are you 99% certain that changes in X3 lead to changes in Y? Are you 99% certain that changes in X4 lead to changes in Y? Be sure to describe the test-statistic used.

b) [25 points] Now assume that you find out that X3 and X4 are highly correlated (ρ = .8). Test the hypothesis that either X3 or X4 are significant at the 99% level? Be sure to describe the test precisely; include equations (and/or inequalities) of the null hypothesis and the research hypothesis. [Big Hint: You estimate the OLS regression of:

$$Y_i \; = \; \beta_{20} \; + \; \beta_{21} * X1_i \; + \; \beta_{22} * X2_i \; + \; v_i \qquad (3)$$

and get an R^2 of 0.83.]

c) [15 points] Which, if any, of the Gaus-Markov assumptions does the multicollinearity of X3 and X4 in part (b) violate?