

Notes on Conditional Relationships and Interactive Terms

To go with Wright, 1976
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Relationships can be *CONDITIONAL*.

Look at the change in relationships across groups, time, geography, etc..

Example: Look at the impact of median family income on the gini coefficient of states. There are two groups of states (North and South), and the distribution of median family income is different across the regions. So, in looking at the effect of income on the gini coefficient one could mistake changes in region for changes in income.

Regression Models:

$$\text{I : } Y = \alpha_{10} + \beta_{11}X + \epsilon_1$$

$$\text{II : } Y = \alpha_{20} + \beta_{21}X + \beta_{22}G^S + \epsilon_2$$

$$\text{III : } Y = \alpha_{30} + \beta_{31}X^S + \beta_{32}X^N + \beta_{33}G^S + \epsilon_3$$

Where:

- α_{20} = intercept for group N
- β_{31} = slope of X for group S
- β_{32} = slope of X for group N
- β_{33} = intercept difference between N and S
- $X^S = X$ for cases in group S ; else 0
- $X^N = X$ for cases in group N ; else 0
- $G^S =$ dummy, 1 if the state is in group S , else 0

Model (3) is really 2 separate regressions, but with the assumption that the two regressions have the same variance ($\text{Var}(\epsilon_1) = \sigma^2$).

We can compare models (2) and (3) using F-tests.

We are testing the restriction:

$$\beta_{31} = \beta_{32}$$

So this is an F-test with 1 linear restriction.

See Wright, Table 3, page 364 for an examination of whether the relationship between income *levels* and income *inequality* is the same for the South and Non-South.

Table 1: Table 3 From Wright - 1976
Gini Index as a Function of Income and Region

	Model		
	I	II	III
Constant	520.7	467.0	459.35
South	–	37.47	97.60
Per Capita Income	-.040	-.018	–
Per Capita Income - South	–	–	-.050
Per Capita Income - North	–	–	-.015
R^2	.389	.597	.615

F tests:

Model I versus Model II: $F(1,47) = 24.31$

$$H_0 : \beta_{22} = 0$$

$$H_1 : \beta_{22} \neq 0$$

Model II versus Model II: $F(1,46) = 2.09$

$$H_0 : \beta_{31} = \beta_{32}$$

$$H_1 : \beta_{31} \neq \beta_{32}$$