Synthesis Techniques

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Synthesis

- It implies the artificial construction of a complex body by combining its elements.
  - Complex body: acoustic signal (sound)
  - Elements: parameters and/or “basic signals”

- Motivations:
  - Reproduce existing sounds
  - Reproduce the physical process of sound generation
  - Generate new pleasant sounds
  - Control/explore timbre
How can I generate new sounds?

- Networks of basic elements → synthesis techniques
- Two main types: linear and non-linear
Additive Synthesis

• It is based on the idea that complex waveforms can be created by the addition of simpler ones.
• It is a linear technique, i.e. do not create frequency components that were not explicitly contained in the original waveforms.
• Commonly, these simpler signals are sinusoids (sines or cosines) with time-varying parameters, according to Fourier’s theory:

\[ s(t) = \sum_{i=0}^{N} A_i \sin(2\pi f_i t + \varphi_i) \]
Additive Synthesis:

A Pipe Organ

HOW A MECHANICAL PIPE ORGAN WORKS

A blower 1 pushes air through a regulating valve into a reservoir 2. From there the air travels up the wind-trunk 3 into an airstream box, the wind-chest 4. A row of pipes is controlled by a stop knob 5. As the knob is pulled out, a wooden plate called a slider 6 is moved, and holes in the slider line up with the pipes. Now these pipes can be played. When the organist depresses a key 7, a pallet 8 opens, and air enters a key channel 9. All the pipes on that channel (whose stops have been opened) will sound.
Additive Synthesis

- Square wave: only odd harmonics. Amplitude of the $n^{th}$ harmonic $= 1/n$
Time-varying sounds

- According to Fourier, all sounds can be described and reproduced with additive synthesis.
- Even impulse-like components can be represented by using a short-lived sinusoid with “infinite” amplitude.

- Additive synthesis is very general (perhaps the most versatile).
- Control data hungry: large number of parameters are required to reproduce realistic sounds
Examples

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Subtractive Synthesis

- Is another linear technique based on the idea that sounds can be generated from subtracting (filtering out) components from a very rich signal (e.g. noise, square wave).

- Its simplicity made it very popular for the design of analog synthesizers (e.g. Moog)
The human speech system

- The vocal chords act as an oscillator, the mouth/nose cavities, tongue and throat as filters.
- We can shape a tonal sound (‘oooh’ vs ‘aaah’), we can whiten the signal (‘ssssshhh’), we can produce pink noise by removing high frequencies.
Source-Filter model

- Subtractive synthesis can be seen as an excitation-resonator or source-filter model.
- The resonator or filter shapes the spectrum, i.e., defines the spectral envelope.
Source-Filter model

Whitening of the signal

Transformations

Analysis

Processing

Synthesis
Amplitude modulation

- Non-linear technique, i.e. results on the creation of frequencies which are not produced by the oscillators.
- In AM the amplitude of the carrier wave is varied in direct proportion to that of a modulating signal.

\[ \text{Amp}_m(t) \rightarrow \text{Amp}_c(t) \]

\[ \text{Freq}_m(t) \rightarrow \text{Freq}_c(t) \]

Bipolar -> Ring modulation
Unipolar -> Amplitude modulation
Ring Modulation

• Let us define the carrier signal as:

\[ c(t) = A_c \cos(\omega_c t) \]

• And the (bipolar) modulator signal as:

\[ m(t) = A_m \cos(\omega_m t) \]

• The Ring modulated signal can be expressed as:

\[ s(t) = A_c \cos(\omega_c t) \cdot A_m \cos(\omega_m t) \]

• Which can be re-written as:

\[ s(t) = \frac{A_c A_m}{2} \left[ \cos([\omega_c - \omega_m]t) + \cos([\omega_c + \omega_m]t) \right] \]

• \( s(t) \) presents two sidebands at frequencies: \( \omega_c - \omega_m \) and \( \omega_c + \omega_m \)
Ring Modulation

![Graph showing ring modulation with carrier signal, modulating signal, and modulated signal](image)

The diagram illustrates the process of ring modulation, where the modulating signal is applied to the carrier signal, resulting in the generation of two new frequencies: $f_c - f_m$ and $f_c + f_m$.
Amplitude Modulation

- Let us define the carrier signal as:

\[ c(t) = \cos(\omega_c t) \]

- And the (unipolar) modulator signal as:

\[ m(t) = A_c + A_m \cos(\omega_m t) \]

- The amplitude modulated signal can be expressed as:

\[ s(t) = \left[ A_c + A_m \cos(\omega_m t) \right] \cos(\omega_c t) \]

- Which can be re-written as:

\[ s(t) = A_c \cos(\omega_c t) + \frac{A_m}{2} \left[ \cos(\omega_c - \omega_m) t + \cos(\omega_c + \omega_m) t \right] \]

- \( s(t) \) presents components at frequencies: \( \omega_c, \omega_c - \omega_m \) and \( \omega_c + \omega_m \)
Modulation index

- In modulation techniques a modulation index is usually defined such that it indicates how much the modulated variable varies around its original value.
- For AM this quantity is also known as modulation depth: \( \beta = \frac{A_m}{A_c} \)
- If \( \beta = 0.5 \) then the carrier’s amplitude varies by 50% around its unmodulated level.
- For \( \beta = 1 \) it varies by 100%.
- \( \beta > 1 \) causes distortion and is usually avoided.
C/M frequency ratio

- Let's define the carrier to modulator frequency ratio \( c/m \) \( (= \omega_c / \omega_m) \) for a pitched signal \( m(t) \)

- If \( c/m \) is an integer \( n \), then \( \omega_c \), and all present frequencies, are multiples of \( \omega_m \) (which will become the fundamental)

- If \( c/m = 1/n \), then \( \omega_c \) will be the fundamental

- When \( c/m \) deviates from \( n \) or \( 1/n \) (or more generally, from a ratio of integers), then the output frequencies becomes more inharmonic

- Example of C/M frequency variation
Frequency Modulation

- Frequency modulation (FM) is a form of modulation in which the frequency of a carrier wave is varied in direct proportion to the amplitude variation of a modulating signal.

- When the frequency modulation produces a variation of less than 20Hz this results on a vibrato.
Frequency Modulation

• Let us define the carrier signal as:

\[ c(t) = \cos(\omega_c t) \]

• And the modulator signal as:

\[ m(t) = \beta \sin(\omega_m t) \]

• The Frequency modulated signal can be expressed as:

\[ s(t) = \cos(\omega_c t + \beta \sin(\omega_m t)) \]

• This can be re-written as

\[ s(t) = \sum_{k=-\infty}^{\infty} J_k(\beta) \cos[(\omega_c + k\omega_m)t] \]
Frequency Modulation

- If $\beta \neq 0$ then the FM spectrum contains infinite sidebands at positions $\omega_c \pm k\omega_m$.

- The amplitudes of each pair of sidebands are given by the $J_k$ coefficients which are functions of $\beta$. 
Modulation index

- As in AM we define a FM modulation index that controls the modulation depth.
- In FM synthesis this index is equal to $\beta$, the amplitude of the modulator and is directly proportional to $\Delta f$.
- As we have seen the value of $\beta$ determines the amplitude of the sidebands of the FM spectrum.
- Furthermore the amplitude decreases with the order $k$.
- Thus, although theoretically the number of sidebands is infinite, in practice their amplitude makes them inaudible for higher orders.
- The number of audible sidebands is a function of $\beta$, and is approximated by $2\beta + 1$.
- Thus the bandwidth increases with the amplitude of $m(t)$, like in some real instruments.
C/M frequency ratio

- The ratio between the carrier and modulator frequencies $c/m$ is relevant to define the (in)harmonic characteristic of $s(t)$.

- The sound is pitched (harmonic) if $c/m$ is a ratio of positive integers: $\omega_c / \omega_m = N_c / N_m$

- E.g. for $f_c = 800$ Hz and $f_m = 200$ Hz, we have sidebands at 600Hz and 1kHz, 400Hz and 1.2kHz, 200Hz and 1.4kHz, etc

- Thus the fundamental frequency of the harmonic spectrum responds to: $f_0 = f_c / N_c = f_m / N_m$

- If $c/m$ is not rational an inharmonic spectrum is produced

- If $f_0$ is below the auditory range, the sound will not be perceived as having definitive pitch.
FM examples

- **A**: c:m ratio 10:1, I = 6
- **B**: c:m ratio 1:1, I = 1
- **C**: c:m ratio 1:10, I = 0.5